6.1 Polygons

What you should learn

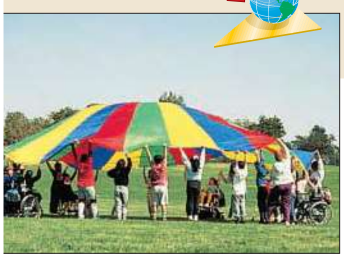
GOAL 1 Identify, name, and describe polygons such as the building shapes in **Example 2**.

GOAL 2 Use the sum of the measures of the interior angles of a quadrilateral.

Why you should learn it

▼ To describe **real-life** objects, such as the parachute in **Exs. 21–23**.

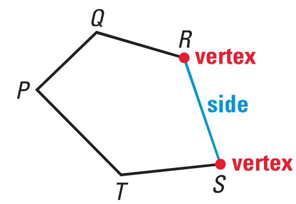
REAL LIFE



GOAL 1 Describing Polygons

A **polygon** is a plane figure that meets the following conditions.

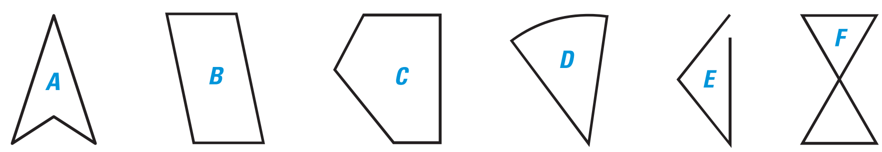
1. It is formed by three or more segments called **sides**, such that no two sides with a common endpoint are collinear.
2. Each side intersects exactly two other sides, one at each endpoint.



Each endpoint of a side is a **vertex** of the polygon. The plural of *vertex* is *vertices*. You can name a polygon by listing its vertices *consecutively*. For instance, *PQRST* and *QPTSR* are two correct names for the polygon above.

1 Identifying Polygons

State whether the figure is a polygon. If it is not, explain why.



Solution

Figures *A*, *B*, and *C* *are* polygons.

- Figure *D is not* a polygon because it has a side that is not a segment.
- Figure *E is not* a polygon because two of the sides intersect only one other side.
- Figure *F is not* a polygon because some of its sides intersect more than two other sides.

. . . .

Polygons are named by the number of sides they have.

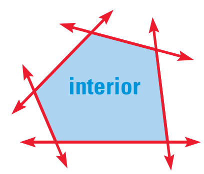
Number of sides	Type of polygon
3	Triangle
4	Quadrilateral
5	Pentagon
6	Hexagon
7	Heptagon

Number of sides	Type of polygon
8	Octagon
9	Nonagon
10	Decagon
12	Dodecagon
n	*n*-gon

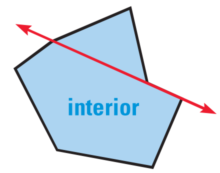
Student Help

Study Tip
To name a polygon not listed in the table, use the number of sides. For example, a polygon with 14 sides is a 14-gon.

A polygon is **convex** if no line that contains a side of the polygon contains a point in the interior of the polygon. A polygon that is not convex is called **nonconvex** or **concave**.



convex polygon



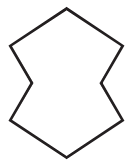
concave polygon

2 *Identifying Convex and Concave Polygons*

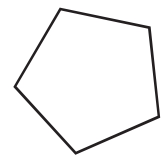

This tile pattern in Iran contains both convex and concave polygons.

Identify the polygon and state whether it is convex or concave.

a.

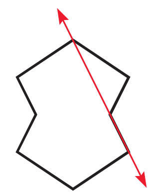


b.

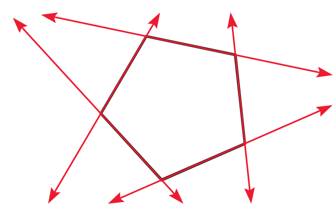


SOLUTION

a. The polygon has 8 sides, so it is an octagon. When extended, some of the sides intersect the interior, so the polygon is concave.



b. The polygon has 5 sides, so it is a pentagon. When extended, none of the sides intersect the interior, so the polygon is convex.

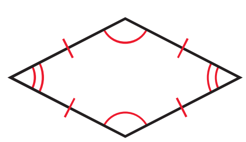


A polygon is **equilateral** if all of its sides are congruent. A polygon is **equiangular** if all of its interior angles are congruent. A polygon is **regular** if it is equilateral and equiangular.

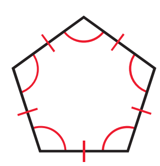
3 *Identifying Regular Polygons*

Decide whether the polygon is regular.

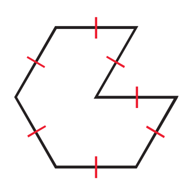
a.



b.



c.



SOLUTION

a. The polygon is an equilateral quadrilateral, but not equiangular. So, it is not a regular polygon.

b. This pentagon is equilateral and equiangular. So, it is a regular polygon.

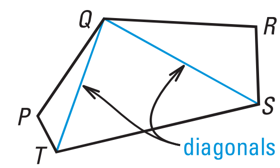
c. This heptagon is equilateral, but not equiangular. So, it is not regular.

GOAL 2 Interior Angles of Quadrilaterals

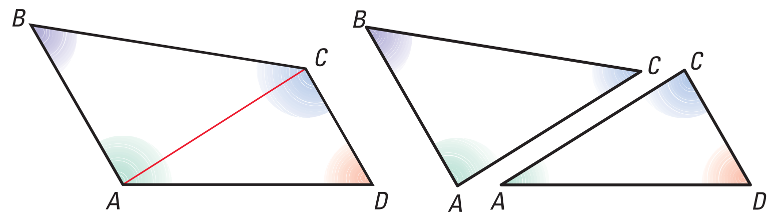
Student Help

Study Tip
Two vertices that are endpoints of the same side are called *consecutive vertices.* For example, *P* and *Q* are consecutive vertices.

A **diagonal** of a polygon is a segment that joins two *nonconsecutive* vertices. Polygon $PQRST$ has 2 diagonals from point Q, $\overline{QT}$ and $\overline{QS}$.



Like triangles, quadrilaterals have both *interior* and *exterior* angles. If you draw a diagonal in a quadrilateral, you divide it into two triangles, each of which has interior angles with measures that add up to 180°. So you can conclude that the sum of the measures of the interior angles of a quadrilateral is 2(180°), or 360°.

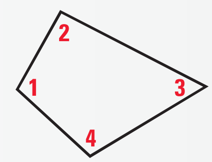


THEOREM

THEOREM 6.1 ***Interior Angles of a Quadrilateral***

The sum of the measures of the interior angles of a quadrilateral is 360°.

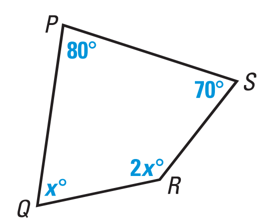
$$m\angle 1 + m\angle 2 + m\angle 3 + m\angle 4 = 360°$$



Example 4 *Interior Angles of a Quadrilateral*

Using Algebra

Find $m\angle Q$ and $m\angle R$.



Solution

Find the value of x. Use the sum of the measures of the interior angles to write an equation involving x. Then, solve the equation.

$x° + 2x° + 70° + 80° = 360°$	Sum of measures of int. $\angle$s of a quad. is 360°.
$3x + 150 = 360$	Combine like terms.
$3x = 210$	Subtract 150 from each side.
$x = 70$	Divide each side by 3.

Find $m\angle Q$ and $m\angle R$.

$m\angle Q = x° = 70°$

$m\angle R = 2x° = 140°$

▶ So, $m\angle Q = 70°$ and $m\angle R = 140°$.

GUIDED PRACTICE

Vocabulary Check ✓

1. What is the plural of *vertex*?

2. What do you call a polygon with 8 sides? a polygon with 15 sides?

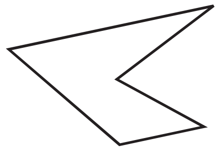
Concept Check ✓

3. Suppose you could tie a string tightly around a convex polygon. Would the length of the string be equal to the perimeter of the polygon? What if the polygon were concave? Explain.

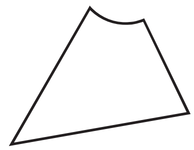
Skill Check ✓

Decide whether the figure is a polygon. If it is not, explain why.

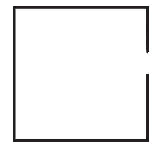
4.



5.

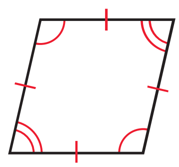


6.

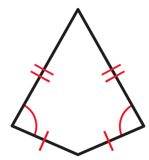


Tell whether the polygon is best described as *equiangular, equilateral, regular,* or *none of these*.

7.



8.



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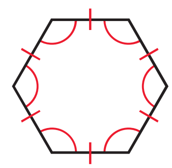
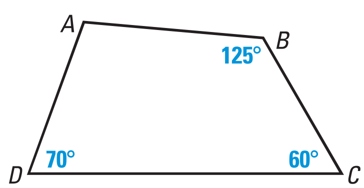
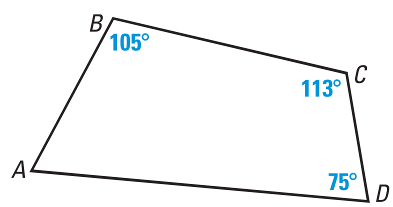


Use the information in the diagram to find $m\angle A$.

10.



11.



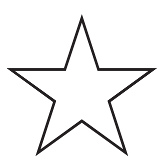
PRACTICE AND APPLICATIONS

STUDENT HELP

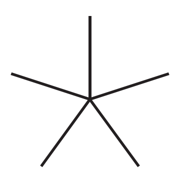
Extra Practice
to help you master skills is on p. 813.

RECOGNIZING POLYGONS **Decide whether the figure is a polygon.**

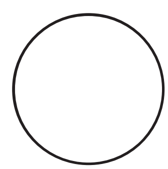
12.



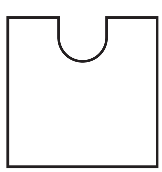
13.



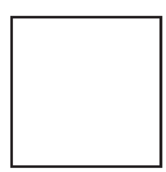
14.



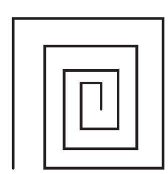
15.



16.



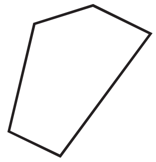
17.

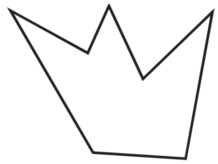


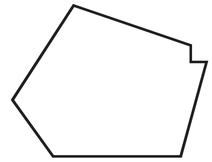
STUDENT HELP

HOMEWORK HELP
Example 1: Exs. 12–17, 48–51
Example 2: Exs. 18–20, 48–51
Example 3: Exs. 24–30, 48–51
Example 4: Exs. 36–46

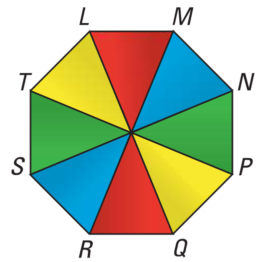
CONVEX OR CONCAVE Use the number of sides to tell what kind of polygon the shape is. Then state whether the polygon is *convex* or *concave.*

18. 

19. 

20. 

PARACHUTES Some gym classes use parachutes that look like the polygon at the right.

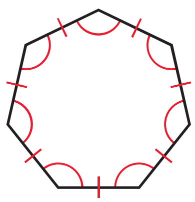


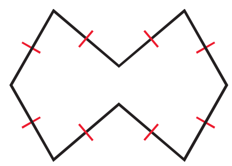
21. Is the polygon a *heptagon*, *octagon*, or *nonagon*?

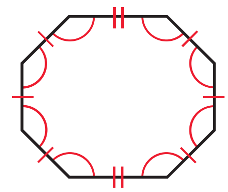
22. Polygon *LMNPQRST* is one name for the polygon. State two other names.

23. Name all of the diagonals that have vertex *M* as an endpoint. Not all of the diagonals are shown.

RECOGNIZING PROPERTIES State whether the polygon is best described as *equilateral, equiangular, regular,* or *none of these*.

24. 

25. 

26. 

TRAFFIC SIGNS Use the number of sides of the traffic sign to tell what kind of polygon it is. Is it *equilateral, equiangular, regular,* or *none of these*?

27. 

28. 

29. 

30. 

FOCUS ON APPLICATIONS

ROAD SIGNS The shape of a sign tells what it is for. For example, triangular signs like the one above are used internationally as warning signs.

DRAWING Draw a figure that fits the description.

31. A convex heptagon

32. A concave nonagon

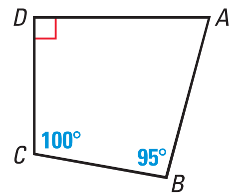
33. An equilateral hexagon that is not equiangular

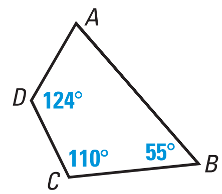
34. An equiangular polygon that is not equilateral

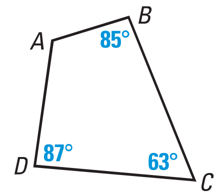
35. **LOGICAL REASONING** Is every triangle convex? Explain your reasoning.

36. **LOGICAL REASONING** Quadrilateral *ABCD* is regular. What is the measure of $\angle ABC$? How do you know?

ANGLE MEASURES **Use the information in the diagram to find $m\angle A$.**

37. 

38. 

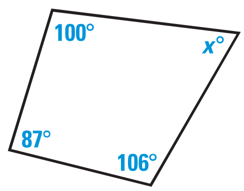
39. 

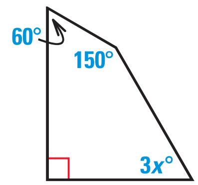
40. **TECHNOLOGY** Use geometry software to draw a quadrilateral. Measure each interior angle and calculate the sum. What happens to the sum as you drag the vertices of the quadrilateral?

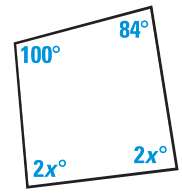
STUDENT HELP

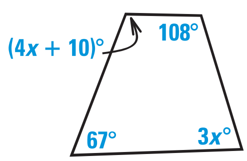
INTERNET SOFTWARE HELP Visit our Web site www.mcdougallittell.com to see instructions for several software applications.

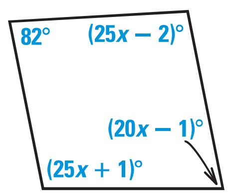
USING ALGEBRA **Use the information in the diagram to solve for x.**

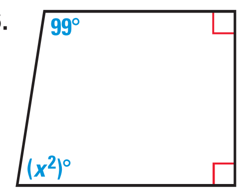
41. 

42. 

43. 

44. 

45. 

46. 

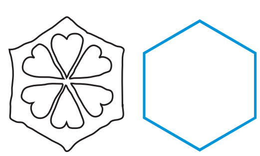
47. **LANGUAGE CONNECTION** A ***deca**gon* has ten sides and a ***deca**de* has ten years. The prefix *deca-* comes from Greek. It means *ten*. What does the prefix *tri-* mean? List four words that use *tri-* and explain what they mean.

PLANT SHAPES **In Exercises 48–51, use the following information.**
Cross sections of seeds and fruits often resemble polygons. Next to each cross section is the polygon it resembles. Describe each polygon. Tell what kind of polygon it is, whether it is *concave* or *convex*, and whether it appears to be *equilateral*, *equiangular*, *regular*, or *none of these*. ▶ Source: The History and Folklore of N. American Wildflowers

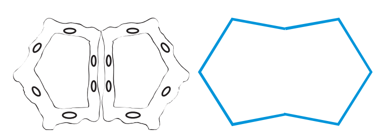
FOCUS ON APPLICATIONS

REAL LIFE **CARAMBOLA**, or star fruit, has a cross section shaped like a 5 pointed star. The fruit comes from an evergreen tree whose leaflets may fold at night or when the tree is shaken.

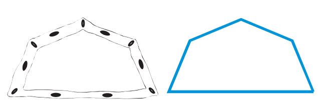
48. Virginia Snakeroot



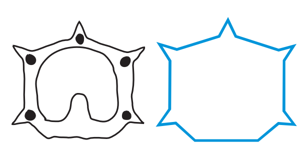
49. Caraway



50. Fennel

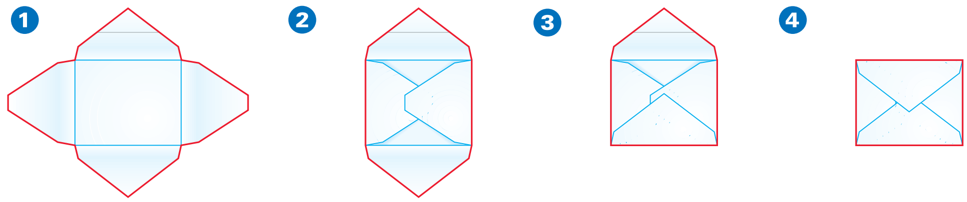


51. Poison Hemlock



Test Preparation

52. **MULTI-STEP PROBLEM** Envelope manufacturers fold a specially-shaped piece of paper to make an envelope, as shown below.



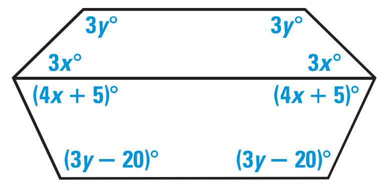
a. What type of polygon is formed by the outside edges of the paper before it is folded? Is the polygon convex?

b. Tell what type of polygon is formed at each step. Which of the polygons are convex?

c. *Writing* Explain the reason for the V-shaped notches that are at the ends of the folds.

★ Challenge

53. **FINDING VARIABLES** Find the values of x and y in the diagram at the right. Check your answer. Then copy the shape and write the measure of each angle on your diagram.

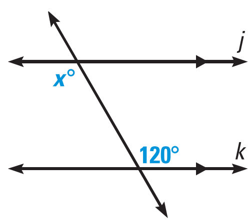


EXTRA CHALLENGE
www.mcdougallittell.com

MIXED REVIEW

PARALLEL LINES **In the diagram, $j \parallel k$. Find the value of x.** (Review 3.3 for 6.2)

54.



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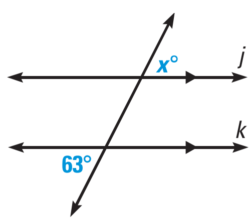

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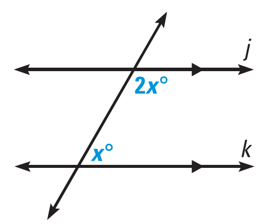

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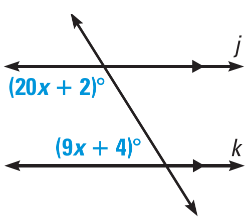

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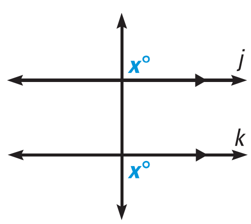

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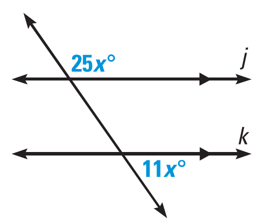


COORDINATE GEOMETRY **You are given the midpoints of the sides of a triangle. Find the coordinates of the vertices of the triangle.** (Review 5.4)

60. $L(-3, 7)$, $M(-5, 1)$, $N(-8, 8)$

61. $L(-4, -1)$, $M(3, 6)$, $N(-2, -8)$

62. $L(2, 4)$, $M(-1, 2)$, $N(0, 7)$

63. $L(-1, 3)$, $M(6, 7)$, $N(3, -5)$

64. **USING ALGEBRA** Use the Distance Formula to find the lengths of the diagonals of a polygon with vertices $A(0, 3)$, $B(3, 3)$, $C(4, 1)$, $D(0, -1)$, and $E(-2, 1)$. (Review 1.3)

6.2 Properties of Parallelograms

What *you should learn*

GOAL 1 Use some properties of parallelograms.

GOAL 2 Use properties of parallelograms in **real-life** situations, such as the drafting table shown in **Example 6**.

Why *you should learn it*

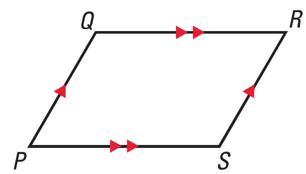
▼ You can use properties of parallelograms to understand how a scissors lift works in **Exs. 51–54**.

REAL LIFE

GOAL 1 PROPERTIES OF PARALLELOGRAMS

In this lesson and in the rest of the chapter you will study special quadrilaterals. A **parallelogram** is a quadrilateral with both pairs of opposite sides parallel.

When you mark diagrams of quadrilaterals, use matching arrowheads to indicate which sides are parallel. For example, in the diagram at the right, $\overline{PQ} \parallel \overline{RS}$ and $\overline{QR} \parallel \overline{SP}$. The symbol $\square PQRS$ is read "parallelogram $PQRS$."

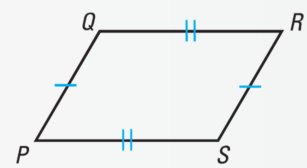


THEOREMS ABOUT PARALLELOGRAMS

THEOREM 6.2

If a quadrilateral is a parallelogram, then its **opposite sides** are congruent.

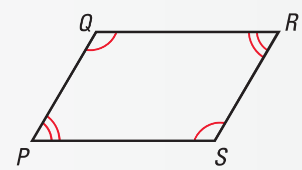
$\overline{PQ} \cong \overline{RS}$ and $\overline{SP} \cong \overline{QR}$



THEOREM 6.3

If a quadrilateral is a parallelogram, then its **opposite angles** are congruent.

$\angle P \cong \angle R$ and $\angle Q \cong \angle S$

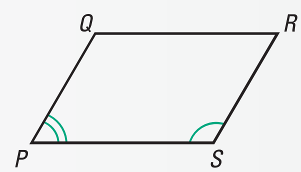


THEOREM 6.4

If a quadrilateral is a parallelogram, then its **consecutive angles** are supplementary.

$m\angle P + m\angle Q = 180°$, $m\angle Q + m\angle R = 180°$,

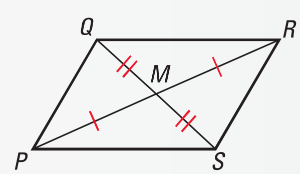
$m\angle R + m\angle S = 180°$, $m\angle S + m\angle P = 180°$



THEOREM 6.5

If a quadrilateral is a parallelogram, then its diagonals bisect each other.

$\overline{QM} \cong \overline{SM}$ and $\overline{PM} \cong \overline{RM}$



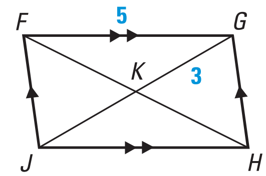
Theorem 6.2 is proved in Example 5. You are asked to prove Theorem 6.3, Theorem 6.4, and Theorem 6.5 in Exercises 38–44.

1 *Using Properties of Parallelograms*

FGHJ is a parallelogram. Find the unknown length. Explain your reasoning.

a. *JH*

b. *JK*



SOLUTION

a. $JH = FG$ — **Opposite sides of a ▱ are $\cong$.**

$JH = 5$ — **Substitute 5 for *FG*.**

b. $JK = GK$ — **Diagonals of a ▱ bisect each other.**

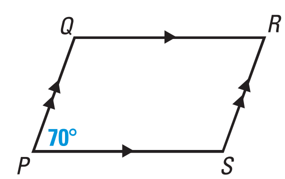
$JK = 3$ — **Substitute 3 for *GK*.**

2 *Using Properties of Parallelograms*

PQRS is a parallelogram. Find the angle measure.

a. $m\angle R$

b. $m\angle Q$



SOLUTION

a. $m\angle R = m\angle P$ — **Opposite angles of a ▱ are $\cong$.**

$m\angle R = 70°$ — **Substitute 70° for $m\angle P$.**

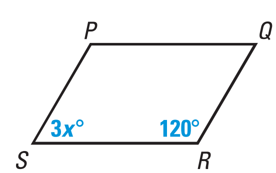
b. $m\angle Q + m\angle P = 180°$ — **Consecutive $\angle$s of a ▱ are supplementary.**

$m\angle Q + 70° = 180°$ — **Substitute 70° for $m\angle P$.**

$m\angle Q = 110°$ — **Subtract 70° from each side.**

3 *Using Algebra with Parallelograms*

PQRS is a parallelogram. Find the value of *x*.



SOLUTION

$m\angle S + m\angle R = 180°$ — **Consecutive angles of a ▱ are supplementary.**

$3x + 120 = 180$ — **Substitute $3x$ for $m\angle S$ and 120 for $m\angle R$.**

$3x = 60$ — **Subtract 120 from each side.**

$x = 20$ — **Divide each side by 3.**

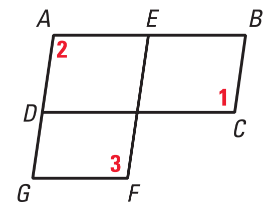
GOAL 2 REASONING ABOUT PARALLELOGRAMS

EXAMPLE 4 *Proving Facts about Parallelograms*

GIVEN ▶ *ABCD* and *AEFG* are parallelograms.

PROVE ▶ $\angle 1 \cong \angle 3$

Plan Show that both angles are congruent to $\angle 2$. Then use the Transitive Property of Congruence.



SOLUTION

Method 1 Write a two-column proof.

Statements	Reasons
1. *ABCD* is a ▱. *AEFG* is a ▱.	**1.** Given
2. $\angle 1 \cong \angle 2$, $\angle 2 \cong \angle 3$	**2.** Opposite angles of a ▱ are $\cong$.
3. $\angle 1 \cong \angle 3$	**3.** Transitive Property of Congruence

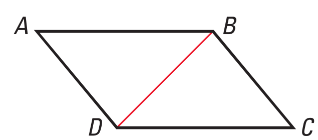
Method 2 Write a paragraph proof.

ABCD is a parallelogram, so $\angle 1 \cong \angle 2$ because opposite angles of a parallelogram are congruent. *AEFG* is a parallelogram, so $\angle 2 \cong \angle 3$. By the Transitive Property of Congruence, $\angle 1 \cong \angle 3$.

EXAMPLE 5 *Proving Theorem 6.2*

GIVEN ▶ *ABCD* is a parallelogram.

PROVE ▶ $\overline{AB} \cong \overline{CD}$, $\overline{AD} \cong \overline{CB}$



SOLUTION

Statements	Reasons
1. *ABCD* is a ▱.	**1.** Given
2. Draw $\overline{BD}$.	**2.** Through any two points there exists exactly one line.
3. $\overline{AB} \parallel \overline{CD}$, $\overline{AD} \parallel \overline{CB}$	**3.** Definition of parallelogram
4. $\angle ABD \cong \angle CDB$, $\angle ADB \cong \angle CBD$	**4.** Alternate Interior Angles Theorem
5. $\overline{DB} \cong \overline{DB}$	**5.** Reflexive Property of Congruence
6. $\triangle ADB \cong \triangle CBD$	**6.** ASA Congruence Postulate
7. $\overline{AB} \cong \overline{CD}$, $\overline{AD} \cong \overline{CB}$	**7.** Corresponding parts of $\cong$ △s are $\cong$.

FOCUS ON CAREERS

FURNITURE DESIGN

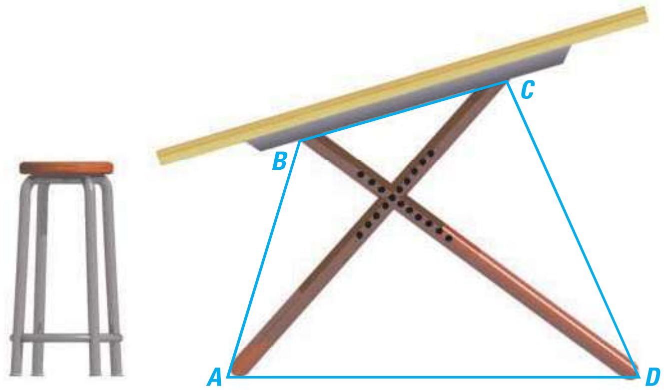
Furniture designers use geometry, trigonometry, and other skills to create designs for furniture.

CAREER LINK

www.mcdougallittell.com

6 *Using Parallelograms in Real Life*

FURNITURE DESIGN A drafting table is made so that the legs can be joined in different ways to change the slope of the drawing surface. In the arrangement below, the legs $\overline{AC}$ and $\overline{BD}$ do *not* bisect each other. Is *ABCD* a parallelogram?



SOLUTION

No. If *ABCD* were a parallelogram, then by Theorem 6.5 $\overline{AC}$ would bisect $\overline{BD}$ and $\overline{BD}$ would bisect $\overline{AC}$.

GUIDED PRACTICE

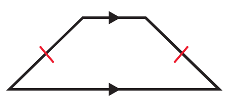
Vocabulary Check ✓

1. Write a definition of *parallelogram*.

Concept Check ✓

Decide whether the figure is a parallelogram. If it is not, explain why not.

2.



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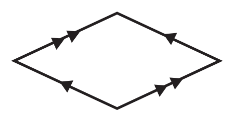


Skill Check ✓

IDENTIFYING CONGRUENT PARTS **Use the diagram of parallelogram *JKLM* at the right. Complete the statement, and give a reason for your answer.**

4. $\overline{JK} \cong$ __?__

5. $\overline{MN} \cong$ __?__

6. $\angle MLK \cong$ __?__

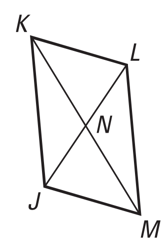
7. $\angle JKL \cong$ __?__

8. $\overline{JN} \cong$ __?__

9. $\overline{KL} \cong$ __?__

10. $\angle MNL \cong$ __?__

11. $\angle MKL \cong$ __?__



Find the measure in parallelogram *LMNQ*. Explain your reasoning.

12. *LM*

13. *LP*

14. *LQ*

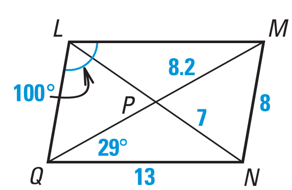
15. *QP*

16. $m\angle LMN$

17. $m\angle NQL$

18. $m\angle MNQ$

19. $m\angle LMQ$

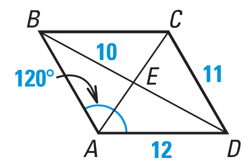


Practice and Applications

Student Help

Extra Practice to help you master skills is on p. 813.

Finding Measures **Find the measure in parallelogram *ABCD*. Explain your reasoning.**



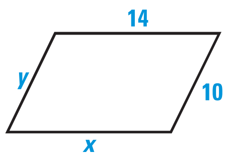
20. DE **21.** BA

22. BC **23.** $m\angle CDA$

24. $m\angle ABC$ **25.** $m\angle BCD$

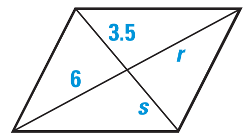
Using Algebra **Find the value of each variable in the parallelogram.**

26.



27.

28.



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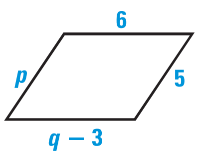

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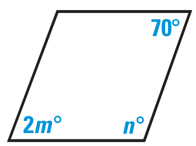

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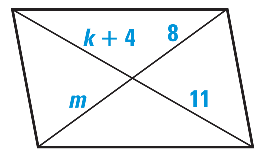
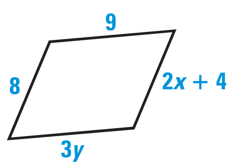


Using Algebra **Find the value of each variable in the parallelogram.**

32.



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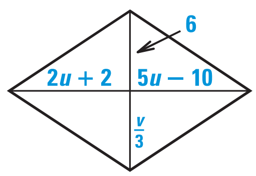

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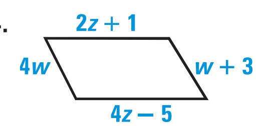

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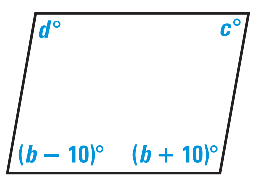

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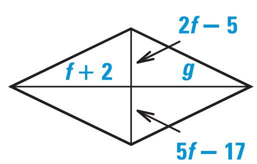

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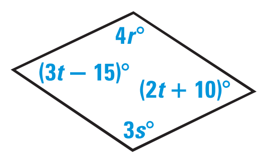
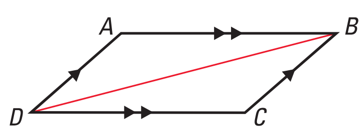


38. **Proving Theorem 6.3** Copy and complete the proof of Theorem 6.3: If a quadrilateral is a parallelogram, then its opposite angles are congruent.

GIVEN ▶ $ABCD$ is a ▱.

PROVE ▶ $\angle A \cong \angle C$, $\angle B \cong \angle D$



Paragraph Proof Opposite sides of a parallelogram are congruent, so ___**a.**___ and ___**b.**___. By the Reflexive Property of Congruence, ___**c.**___. $\triangle ABD \cong \triangle CDB$ because of the ___**d.**___ Congruence Postulate. Because ___**e.**___ parts of congruent triangles are congruent, $\angle A \cong \angle C$.

To prove that $\angle B \cong \angle D$, draw ___**f.**___ and use the same reasoning.

Student Help

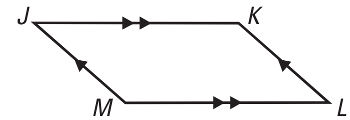
HOMEWORK HELP

Example 1: Exs. 20–22
Example 2: Exs. 23–25
Example 3: Exs. 26–37
Example 4: Exs. 55–58
Example 5: Exs. 38–44
Example 6: Exs. 45–54

39. **PROVING THEOREM 6.4** Copy and complete the two-column proof of Theorem 6.4: If a quadrilateral is a parallelogram, then its consecutive angles are supplementary.

GIVEN ▶ $JKLM$ is a ▱.

PROVE ▶ $\angle J$ and $\angle K$ are supplementary.



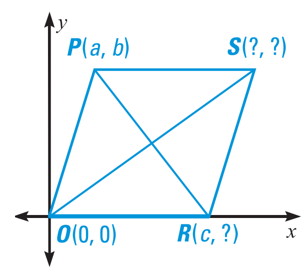
Statements	Reasons
1. ___?___	**1.** Given
2. $m\angle J = m\angle L$, $m\angle K = m\angle M$	**2.** ___?___
3. $m\angle J + m\angle L + m\angle K + m\angle M =$ ___?___	**3.** Sum of measures of int. ∠s of a quad. is 360°.
4. $m\angle J + m\angle J + m\angle K + m\angle K = 360°$	**4.** ___?___
5. 2(___?___ + ___?___) = 360°	**5.** Distributive property
6. $m\angle J + m\angle K = 180°$	**6.** ___?___ prop. of equality
7. $\angle J$ and $\angle K$ are supplementary.	**7.** ___?___

You can use the same reasoning to prove any other pair of consecutive angles in ▱$JKLM$ are supplementary.

DEVELOPING COORDINATE PROOF **Copy and complete the coordinate proof of Theorem 6.5.**

GIVEN ▶ $PORS$ is a ▱.

PROVE ▶ $\overline{PR}$ and $\overline{OS}$ bisect each other.



Plan for Proof Find the coordinates of the midpoints of the diagonals of ▱$PORS$ and show that they are the same.

STUDENT HELP

INTERNET HOMEWORK HELP Visit our Web site www.mcdougallittell.com for help with the coordinate proof in Exs. 40–44.

40. Point R is on the x-axis, and the length of $\overline{OR}$ is c units. What are the coordinates of point R?

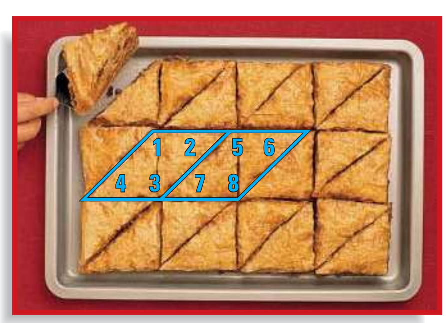
41. The length of $\overline{PS}$ is also c units, and $\overline{PS}$ is horizontal. What are the coordinates of point S?

42. What are the coordinates of the midpoint of $\overline{PR}$?

43. What are the coordinates of the midpoint of $\overline{OS}$?

44. *Writing* How do you know that $\overline{PR}$ and $\overline{OS}$ bisect each other?

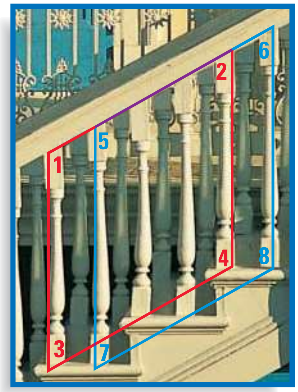
BAKING **In Exercises 45 and 46, use the following information.**
In a recipe for baklava, the pastry should be cut into triangles that form congruent parallelograms, as shown. Write a paragraph proof to prove the statement.



45. $\angle 3$ is supplementary to $\angle 6$.

46. $\angle 4$ is supplementary to $\angle 5$.

STAIR BALUSTERS **In Exercises 47–50, use the following information.** In the diagram at the right, the slope of the handrail is equal to the slope of the stairs. The balusters (vertical posts) support the handrail.



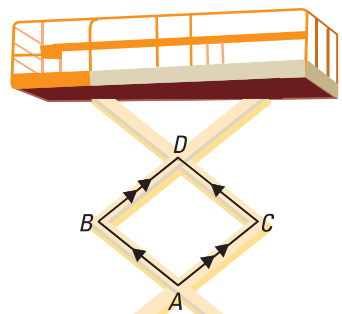
47. Which angle in the red parallelogram is congruent to $\angle 1$?

48. Which angles in the blue parallelogram are supplementary to $\angle 6$?

49. Which postulate can be used to prove that $\angle 1 \cong \angle 5$?

50. *Writing* Is the red parallelogram congruent to the blue parallelogram? Explain your reasoning.

SCISSORS LIFT **Photographers can use scissors lifts for overhead shots, as shown at the left. The crossing beams of the lift form parallelograms that move together to raise and lower the platform. In Exercises 51–54, use the diagram of parallelogram *ABDC* at the right.**



51. What is $m\angle B$ when $m\angle A = 120°$?

52. Suppose you decrease $m\angle A$. What happens to $m\angle B$?

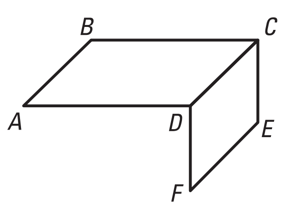
53. Suppose you decrease $m\angle A$. What happens to AD?

54. Suppose you decrease $m\angle A$. What happens to the overall height of the scissors lift?

TWO-COLUMN PROOF **Write a two-column proof.**

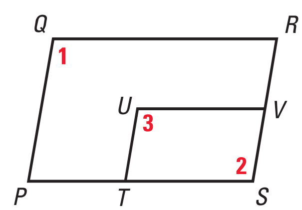
55. GIVEN ▶ *ABCD* and *CEFD* are ▱s.

PROVE ▶ $\overline{AB} \cong \overline{FE}$



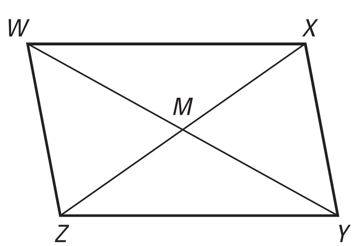
56. GIVEN ▶ *PQRS* and *TUVS* are ▱s.

PROVE ▶ $\angle 1 \cong \angle 3$



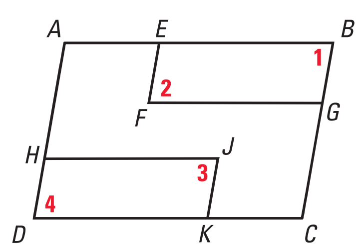
57. GIVEN ▶ *WXYZ* is a ▱.

PROVE ▶ $\triangle WMZ \cong \triangle YMX$

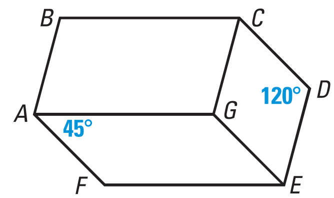


58. GIVEN ▶ *ABCD*, *EBGF*, *HJKD* are ▱s.

PROVE ▶ $\angle 2 \cong \angle 3$



59. *Writing* In the diagram, $ABCG$, $CDEG$, and $AGEF$ are parallelograms. Copy the diagram and add as many other angle measures as you can. Then describe how you know the angle measures you added are correct.

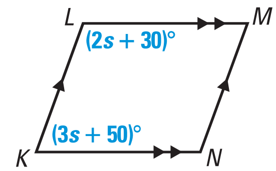


Test Preparation

60. **MULTIPLE CHOICE** In $\square KLMN$, what is the value of s?

Ⓐ 5 Ⓑ 20 Ⓒ 40

Ⓓ 52 Ⓔ 70



61. **MULTIPLE CHOICE** In $\square ABCD$, point E is the intersection of the diagonals. Which of the following is *not* necessarily true?

Ⓐ $AB = CD$ Ⓑ $AC = BD$ Ⓒ $AE = CE$ Ⓓ $AD = BC$ Ⓔ $DE = BE$

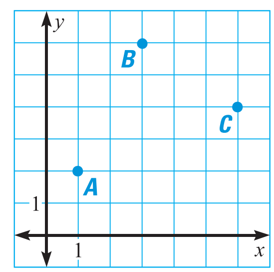
★ Challenge

USING ALGEBRA Suppose points $A(1, 2)$, $B(3, 6)$, and $C(6, 4)$ are three vertices of a parallelogram.

62. Give the coordinates of a point that could be the fourth vertex. Sketch the parallelogram in a coordinate plane.

63. Explain how to check to make sure the figure you drew in Exercise 62 is a parallelogram.

64. How many different parallelograms can be formed using A, B, and C as vertices? Sketch each parallelogram and label the coordinates of the fourth vertex.



EXTRA CHALLENGE

www.mcdougallittell.com

MIXED REVIEW

USING ALGEBRA Use the Distance Formula to find AB. **(Review 1.3 for 6.3)**

65. $A(2, 1)$, $B(6, 9)$

66. $A(-4, 2)$, $B(2, -1)$

67. $A(-8, -4)$, $B(-1, -3)$

USING ALGEBRA Find the slope of $\overline{AB}$. **(Review 3.6 for 6.3)**

68. $A(2, 1)$, $B(6, 9)$

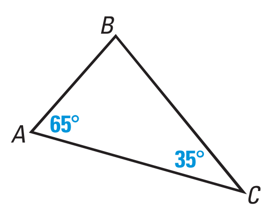
69. $A(-4, 2)$, $B(2, -1)$

70. $A(-8, -4)$, $B(-1, -3)$

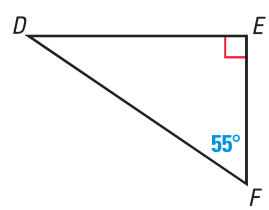
71. **PARKING CARS** In a parking lot, two guidelines are painted so that they are both perpendicular to the line along the curb. Are the guidelines parallel? Explain why or why not. **(Review 3.5)**

Name the shortest and longest sides of the triangle. Explain. **(Review 5.5)**

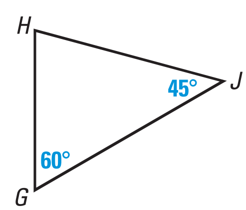
72.



73.



74.



6.3 Proving Quadrilaterals are Parallelograms

What you should learn

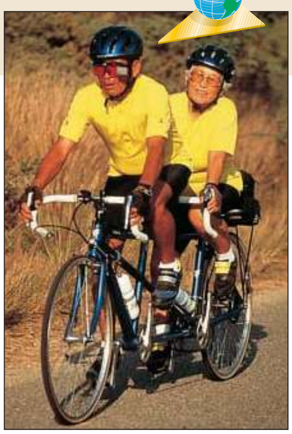
GOAL 1 Prove that a quadrilateral is a parallelogram.

GOAL 2 Use coordinate geometry with parallelograms.

Why you should learn it

▼ To understand how **real-life** tools work, such as the bicycle derailleur in **Ex. 27**, which lets you change gears when you are biking uphill.

REAL LIFE



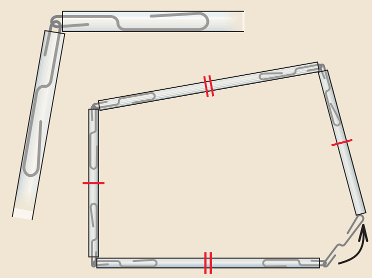
GOAL 1 Proving Quadrilaterals are Parallelograms

The activity illustrates one way to prove that a quadrilateral is a parallelogram.

ACTIVITY Developing Concepts

Investigating Properties of Parallelograms

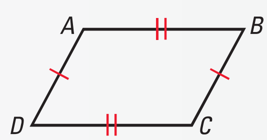
1. Cut four straws to form two congruent pairs.
2. Partly unbend two paperclips, link their smaller ends, and insert the larger ends into two cut straws, as shown. Join the rest of the straws to form a quadrilateral with opposite sides congruent, as shown.
3. Change the angles of your quadrilateral. Is your quadrilateral always a parallelogram?



THEOREMS

THEOREM 6.6

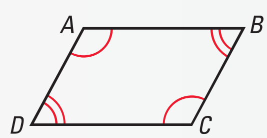
If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram.



***ABCD* is a parallelogram.**

THEOREM 6.7

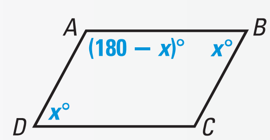
If both pairs of opposite angles of a quadrilateral are congruent, then the quadrilateral is a parallelogram.



***ABCD* is a parallelogram.**

THEOREM 6.8

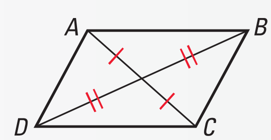
If an angle of a quadrilateral is supplementary to both of its consecutive angles, then the quadrilateral is a parallelogram.



***ABCD* is a parallelogram.**

THEOREM 6.9

If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.



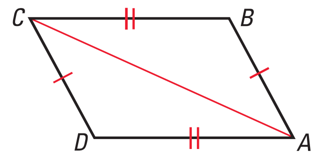
***ABCD* is a parallelogram.**

The proof of Theorem 6.6 is given in Example 1. You will be asked to prove Theorem 6.7, Theorem 6.8, and Theorem 6.9 in Exercises 32–36.

EXAMPLE 1 Proof of Theorem 6.6

Proof

Prove Theorem 6.6.



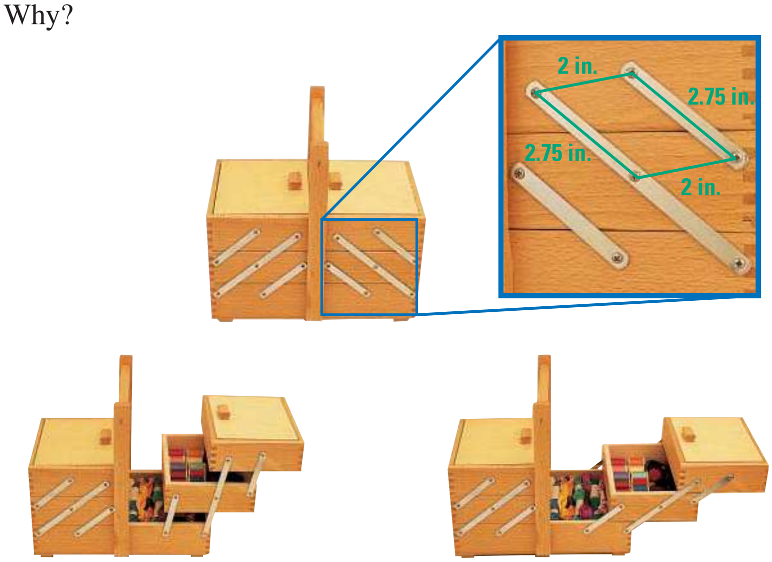
GIVEN ▶ $\overline{AB} \cong \overline{CD}$, $\overline{AD} \cong \overline{CB}$

PROVE ▶ $ABCD$ is a parallelogram.

Statements	Reasons
1. $\overline{AB} \cong \overline{CD}$, $\overline{AD} \cong \overline{CB}$	**1.** Given
2. $\overline{AC} \cong \overline{AC}$	**2.** Reflexive Property of Congruence
3. $\triangle ABC \cong \triangle CDA$	**3.** SSS Congruence Postulate
4. $\angle BAC \cong \angle DCA$, $\angle DAC \cong \angle BCA$	**4.** Corresponding parts of $\cong$ $\triangle$s are $\cong$.
5. $\overline{AB} \parallel \overline{CD}$, $\overline{AD} \parallel \overline{CB}$	**5.** Alternate Interior Angles Converse
6. $ABCD$ is a $\square$.	**6.** Definition of parallelogram

EXAMPLE 2 Proving Quadrilaterals are Parallelograms

As the sewing box below is opened, the trays are always parallel to each other. Why?



SOLUTION

Each pair of hinges are opposite sides of a quadrilateral. The 2.75 inch sides of the quadrilateral are opposite and congruent. The 2 inch sides are also opposite and congruent. Because opposite sides of the quadrilateral are congruent, it is a parallelogram. By the definition of a parallelogram, opposite sides are parallel, so the trays of the sewing box are always parallel.

FOCUS ON APPLICATIONS

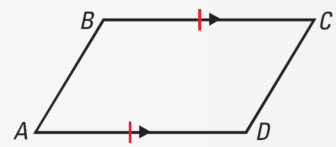
CONTAINERS Many containers, such as tackle boxes, jewelry boxes, and tool boxes, use parallelograms in their design to ensure that the trays stay level.

Theorem 6.10 gives another way to prove a quadrilateral is a parallelogram.

THEOREM

THEOREM 6.10

If one pair of opposite sides of a quadrilateral are congruent and parallel, then the quadrilateral is a parallelogram.

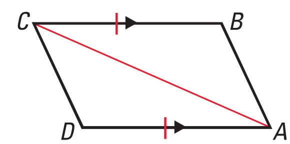


***ABCD* is a parallelogram.**

3 *Proof of Theorem 6.10*

Proof

Prove Theorem 6.10.



GIVEN ▶ $\overline{BC} \parallel \overline{DA}$, $\overline{BC} \cong \overline{DA}$

PROVE ▶ $ABCD$ is a parallelogram.

Plan for Proof Show that $\triangle BAC \cong \triangle DCA$, so $\overline{AB} \cong \overline{CD}$. Use Theorem 6.6.

$\overline{BC} \parallel \overline{DA}$ → $\angle DAC \cong \angle BCA$ → $\triangle BAC \cong \triangle DCA$

Given; Alt. Int. ∠s Thm.; SAS Congruence Post.

$\overline{AC} \cong \overline{AC}$

Refl. Prop. of Cong.

$\overline{BC} \cong \overline{DA}$

Given

$\overline{AB} \cong \overline{CD}$

Corresp. parts of ≅ △ are ≅.

$ABCD$ is a ▱.

If opp. sides of a quad. are ≅, then it is a ▱.

You have studied several ways to prove that a quadrilateral is a parallelogram. In the box below, the first way is also the definition of a parallelogram.

CONCEPT SUMMARY **PROVING QUADRILATERALS ARE PARALLELOGRAMS**

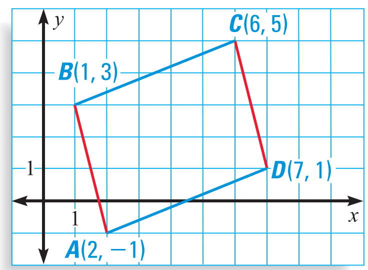
- Show that both pairs of opposite sides are parallel.
- Show that both pairs of opposite sides are congruent.
- Show that both pairs of opposite angles are congruent.
- Show that one angle is supplementary to both consecutive angles.
- Show that the diagonals bisect each other.
- Show that one pair of opposite sides are congruent and parallel.

GOAL 2 USING COORDINATE GEOMETRY

When a figure is in the coordinate plane, you can use the Distance Formula to prove that sides are congruent and you can use the slope formula to prove that sides are parallel.

EXAMPLE 4 *Using Properties of Parallelograms*

Show that $A(2, -1)$, $B(1, 3)$, $C(6, 5)$, and $D(7, 1)$ are the vertices of a parallelogram.



SOLUTION

There are many ways to solve this problem.

Method 1 Show that opposite sides have the same slope, so they are parallel.

$$\text{Slope of } \overline{AB} = \frac{3-(-1)}{1-2} = -4$$

$$\text{Slope of } \overline{CD} = \frac{1-5}{7-6} = -4$$

$$\text{Slope of } \overline{BC} = \frac{5-3}{6-1} = \frac{2}{5}$$

$$\text{Slope of } \overline{DA} = \frac{-1-1}{2-7} = \frac{2}{5}$$

$\overline{AB}$ and $\overline{CD}$ have the same slope so they are parallel. Similarly, $\overline{BC} \parallel \overline{DA}$.

▶ Because opposite sides are parallel, *ABCD* is a parallelogram.

Method 2 Show that opposite sides have the same length.

$$AB = \sqrt{(1-2)^2 + [3-(-1)]^2} = \sqrt{17}$$

$$CD = \sqrt{(7-6)^2 + (1-5)^2} = \sqrt{17}$$

$$BC = \sqrt{(6-1)^2 + (5-3)^2} = \sqrt{29}$$

$$DA = \sqrt{(2-7)^2 + (-1-1)^2} = \sqrt{29}$$

▶ $\overline{AB} \cong \overline{CD}$ and $\overline{BC} \cong \overline{DA}$. Because both pairs of opposite sides are congruent, *ABCD* is a parallelogram.

Method 3 Show that one pair of opposite sides is congruent and parallel.

Find the slopes and lengths of $\overline{AB}$ and $\overline{CD}$ as shown in Methods 1 and 2.

$$\text{Slope of } \overline{AB} = \text{Slope of } \overline{CD} = -4$$

$$AB = CD = \sqrt{17}$$

▶ $\overline{AB}$ and $\overline{CD}$ are congruent and parallel, so *ABCD* is a parallelogram.

STUDENT HELP

Study Tip
Because you don't know the measures of the angles of *ABCD*, you can *not* use Theorems 6.7 or 6.8 in Example 4.

STUDENT HELP

INTERNET
HOMEWORK HELP
Visit our Web site www.mcdougallittell.com for extra examples.

GUIDED PRACTICE

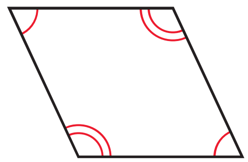
Concept Check ✓

1. Is a hexagon with opposite sides parallel called a parallelogram? Explain.

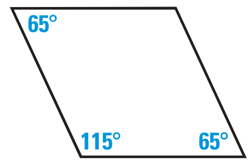
Skill Check ✓

Decide whether you are given enough information to determine that the quadrilateral is a parallelogram. Explain your reasoning.

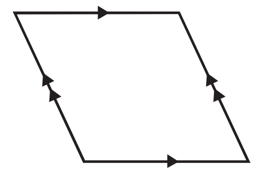
2.



3.

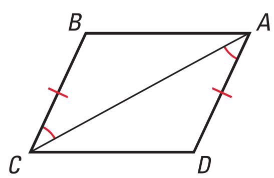


4.

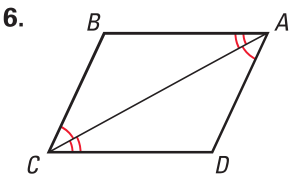


Describe how you would prove that *ABCD* is a parallelogram.

5.



6.



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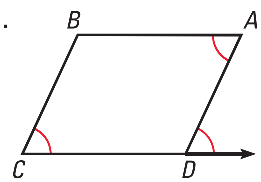


8. Describe at least three ways to show that $A(0, 0)$, $B(2, 6)$, $C(5, 7)$, and $D(3, 1)$ are the vertices of a parallelogram.

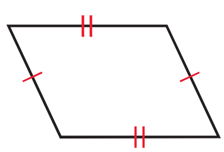
PRACTICE AND APPLICATIONS

STUDENT HELP

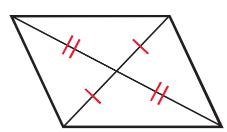
Extra Practice to help you master skills is on p. 813.

LOGICAL REASONING **Are you given enough information to determine whether the quadrilateral is a parallelogram? Explain.**

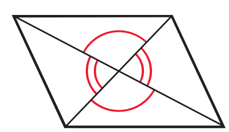
9.



10.



11.



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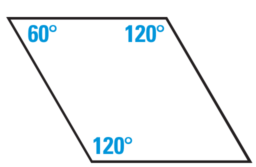

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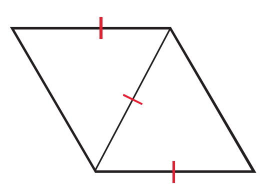

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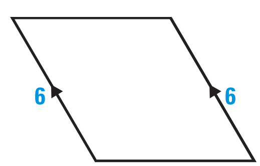


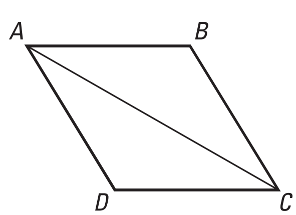
STUDENT HELP

HOMEWORK HELP

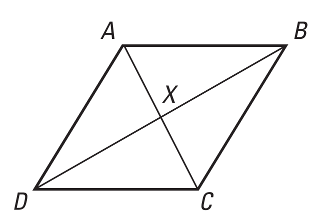
Example 1: Exs. 15, 16, 32, 33
Example 2: Exs. 21, 28, 31
Example 3: Exs. 32, 33
Example 4: Exs. 21–26, 34–36

LOGICAL REASONING **Describe how to prove that *ABCD* is a parallelogram. Use the given information.**

15. $\triangle ABC \cong \triangle CDA$

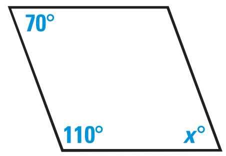


16. $\triangle AXB \cong \triangle CXD$

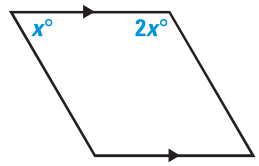


USING ALGEBRA **What value of x will make the polygon a parallelogram?**

17.



18.



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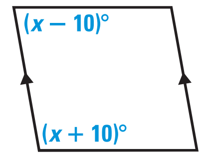


20. **VISUAL THINKING** Draw a quadrilateral that has one pair of congruent sides and one pair of parallel sides but that is not a parallelogram.

COORDINATE GEOMETRY **Use the given definition or theorem to prove that *ABCD* is a parallelogram. Use $A(-1, 6)$, $B(3, 5)$, $C(5, -3)$, and $D(1, -2)$.**

21. Theorem 6.6

22. Theorem 6.9

23. definition of a parallelogram

24. Theorem 6.10

USING COORDINATE GEOMETRY **Prove that the points represent the vertices of a parallelogram. Use a different method for each exercise.**

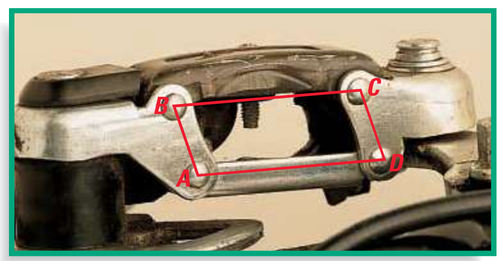
25. $J(-6, 2)$, $K(-1, 3)$, $L(2, -3)$, $M(-3, -4)$

26. $P(2, 5)$, $Q(8, 4)$, $R(9, -4)$, $S(3, -3)$

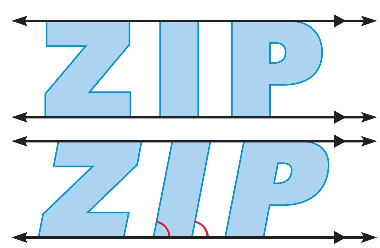
FOCUS ON APPLICATIONS

REAL LIFE **DERAILLEURS** (named from the French word meaning 'derail') move the chain among two to six sprockets of different diameters to change gears.

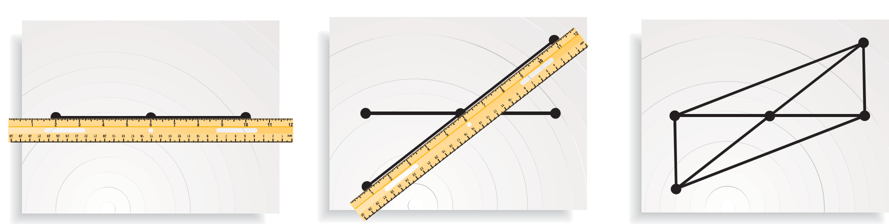
27. **CHANGING GEARS** When you change gears on a bicycle, the *derailleur* moves the chain to the new gear. For the derailleur at the right, $AB = 1.8$ cm, $BC = 3.6$ cm, $CD = 1.8$ cm, and $DA = 3.6$ cm. Explain why $\overline{AB}$ and $\overline{CD}$ are always parallel when the derailleur moves.



28. **COMPUTERS** Many word processors have a feature that allows a regular letter to be changed to an oblique (slanted) letter. The diagram at the right shows some regular letters and their oblique versions. Explain how you can prove that the oblique I is a parallelogram.



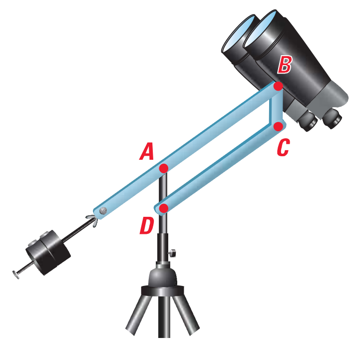
29. **VISUAL REASONING** Explain why the following method of drawing a parallelogram works. State a theorem to support your answer.



1. Use a ruler to draw a segment and its midpoint.
2. Draw another segment so the midpoints coincide.
3. Connect the endpoints of the segments.

30. **CONSTRUCTION** There are many ways to use a compass and straightedge to construct a parallelogram. Describe a method that uses Theorem 6.6, Theorem 6.8, or Theorem 6.10. Then use your method to construct a parallelogram.

31. **BIRD WATCHING** You are designing a binocular mount that will keep the binoculars pointed in the same direction while they are raised and lowered for different viewers. If $\overline{BC}$ is always vertical, the binoculars will always point in the same direction. How can you design the mount so $\overline{BC}$ is always vertical? Justify your answer.



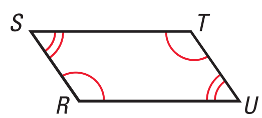
PROVING THEOREMS 6.7 AND 6.8 **Write a proof of the theorem.**

32. Prove Theorem 6.7.

GIVEN ▶ $\angle R \cong \angle T$, $\angle S \cong \angle U$

PROVE ▶ $RSTU$ is a parallelogram.

Plan for Proof Show that the sum $2(m\angle S) + 2(m\angle T) = 360°$, so $\angle S$ and $\angle T$ are supplementary and $\overline{SR} \parallel \overline{UT}$.

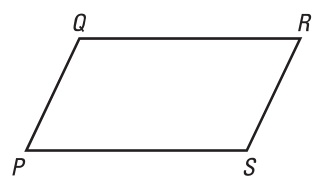


33. Prove Theorem 6.8.

GIVEN ▶ $\angle P$ is supplementary to $\angle Q$ and $\angle S$.

PROVE ▶ $PQRS$ is a parallelogram.

Plan for Proof Show that opposite sides of $PQRS$ are parallel.



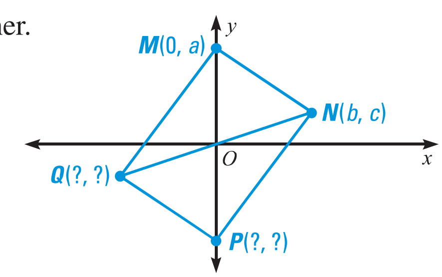
PROVING THEOREM 6.9 **In Exercises 34–36, complete the coordinate proof of Theorem 6.9.**

GIVEN ▶ Diagonals $\overline{MP}$ and $\overline{NQ}$ bisect each other.

PROVE ▶ $MNPQ$ is a parallelogram.

Plan for Proof Show that opposite sides of $MNPQ$ have the same slope.

Place $MNPQ$ in the coordinate plane so the diagonals intersect at the origin and $\overline{MP}$ lies on the y-axis. Let the coordinates of M be $(0, a)$ and the coordinates of N be (b, c). Copy the graph at the right.



34. What are the coordinates of P? Explain your reasoning and label the coordinates on your graph.

35. What are the coordinates of Q? Explain your reasoning and label the coordinates on your graph.

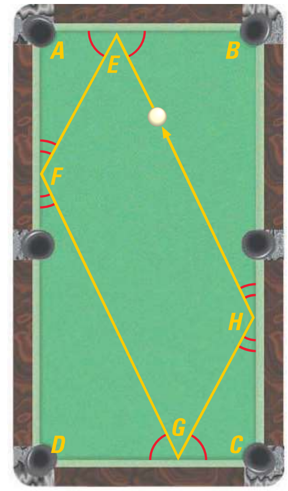
36. Find the slope of each side of $MNPQ$ and show that the slopes of opposite sides are equal.

STUDENT HELP

HOMEWORK HELP
Visit our Web site www.mcdougallittell.com for help with the coordinate proof in Exs. 34–36.

Test Preparation

37. MULTI-STEP PROBLEM You shoot a pool ball as shown at the right and it rolls back to where it started. The ball bounces off each wall at the same angle at which it hit the wall. Copy the diagram and add each angle measure as you know it.



a. The ball hits the first wall at an angle of about $63°$. So $m\angle AEF = m\angle BEH \approx 63°$. Explain why $m\angle AFE \approx 27°$.

b. Explain why $m\angle FGD \approx 63°$.

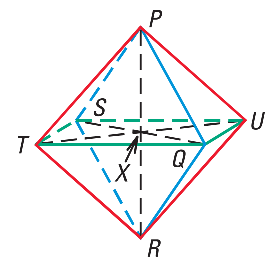
c. What is $m\angle GHC$? $m\angle EHB$?

d. Find the measure of each interior angle of *EFGH*. What kind of shape is *EFGH*? How do you know?

★ **Challenge**

EXTRA CHALLENGE
www.mcdougallittell.com

38. VISUAL THINKING *PQRS* is a parallelogram and *QTSU* is a parallelogram. Use the diagonals of the parallelograms to explain why *PTRU* is a parallelogram.



MIXED REVIEW

USING ALGEBRA Rewrite the biconditional statement as a conditional statement and its converse. (Review 2.2 for 6.4)

39. $x^2 + 2 = 2$ if and only if $x = 0$.

40. $4x + 7 = x + 37$ if and only if $x = 10$.

41. A quadrilateral is a parallelogram if and only if each pair of opposite sides are parallel.

WRITING BICONDITIONAL STATEMENTS Write the pair of theorems from Lesson 5.1 as a single biconditional statement. (Review 2.2, 5.1 for 6.4)

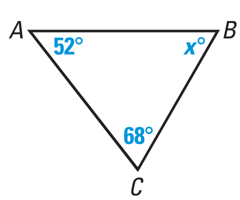
42. Theorems 5.1 and 5.2

43. Theorems 5.3 and 5.4

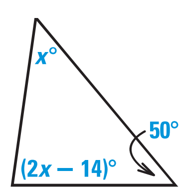
44. Write an equation of the line that is perpendicular to $y = -4x + 2$ and passes through the point $(1, -2)$. (Review 3.7)

ANGLE MEASURES Find the value of *x*. (Review 4.1)

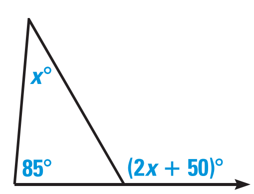
45.



46.



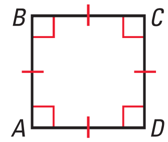
47.



QUIZ 1

Self-Test for Lessons 6.1–6.3

1. Choose the words that describe the quadrilateral at the right: *concave*, *convex*, *equilateral*, *equiangular*, and *regular*. **(Lesson 6.1)**

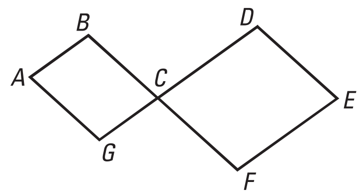


2. Find the value of x. Explain your reasoning. **(Lesson 6.1)**

3. Write a proof. **(Lesson 6.2)**

GIVEN ▶ $ABCG$ and $CDEF$ are parallelograms.

PROVE ▶ $\angle A \cong \angle E$



4. Describe two ways to show that $A(-4, 1)$, $B(3, 0)$, $C(5, -7)$, and $D(-2, -6)$ are the vertices of a parallelogram. **(Lesson 6.3)**

MATH & History

History of Finding Area

APPLICATION LINK
www.mcdougallittell.com

THEN

THOUSANDS OF YEARS AGO, the Egyptians needed to find the area of the land they were farming. The mathematical methods they used are described in a papyrus dating from about 1650 B.C.

NOW

TODAY, satellites and aerial photographs can be used to measure the areas of large or inaccessible regions.

1. Find the area of the trapezoid outlined on the aerial photograph. The formula for the area of a trapezoid appears on page 374.

c.1650 B.C.

This Egyptian papyrus includes methods for finding area.

Methods for finding area are recorded in this Chinese manuscript.

c. 300 B.C.– A.D. 200

Surveyors use signals from satellites to measure large areas.

1990s

6.4 Rhombuses, Rectangles, and Squares

What you should learn

GOAL 1 Use properties of sides and angles of rhombuses, rectangles, and squares.

GOAL 2 Use properties of diagonals of rhombuses, rectangles, and squares.

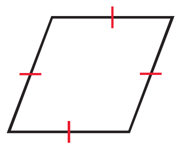
Why you should learn it

▼ To simplify **real-life** tasks, such as checking whether a theater flat is rectangular in **Example 6**.

REAL LIFE

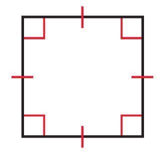
GOAL 1 PROPERTIES OF SPECIAL PARALLELOGRAMS

In this lesson you will study three special types of parallelograms: *rhombuses*, *rectangles*, and *squares*.



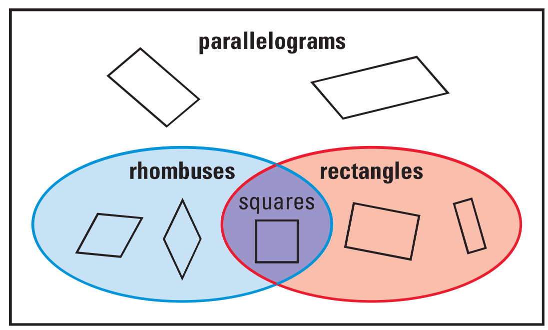
A **rhombus** is a parallelogram with four congruent sides.

A **rectangle** is a parallelogram with four right angles.



A **square** is a parallelogram with four congruent sides and four right angles.

The *Venn diagram* at the right shows the relationships among parallelograms, rhombuses, rectangles, and squares. Each shape has the properties of every group that it belongs to. For instance, a square is a rectangle, a rhombus, and a parallelogram, so it has all of the properties of each of those shapes.

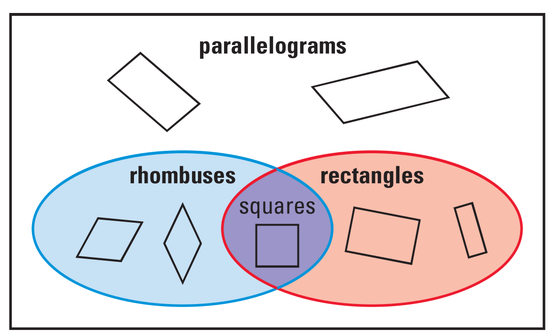


EXAMPLE 1 *Describing a Special Parallelogram*

Decide whether the statement is *always*, *sometimes*, or *never* true.

a. A rhombus is a rectangle.

b. A parallelogram is a rectangle.



SOLUTION

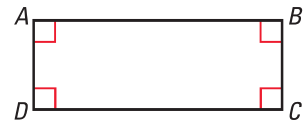
a. The statement is *sometimes* true. In the Venn diagram, the regions for rhombuses and rectangles overlap. If the rhombus is a square, it is a rectangle.

b. The statement is *sometimes* true. Some parallelograms are rectangles. In the Venn diagram, you can see that some of the shapes in the parallelogram box are in the region for rectangles, but many aren't.

Logical Reasoning

GOAL 2 Using Properties of Special Parallelograms

ABCD is a rectangle. What else do you know about *ABCD*?



SOLUTION

Because *ABCD* is a rectangle, it has four right angles by the definition. The definition also states that rectangles are parallelograms, so *ABCD* has all the properties of a parallelogram:

- Opposite sides are parallel and congruent.
- Opposite angles are congruent and consecutive angles are supplementary.
- Diagonals bisect each other.

. . . .

A rectangle is defined as a *parallelogram* with four right angles. But *any quadrilateral* with four right angles is a rectangle because any quadrilateral with four right angles is a parallelogram. In Exercises 48–50 you will justify the following corollaries to the definitions of rhombus, rectangle, and square.

STUDENT HELP

Look Back
For help with biconditional statements, see p. 80.

COROLLARIES ABOUT SPECIAL QUADRILATERALS

RHOMBUS COROLLARY

A quadrilateral is a rhombus if and only if it has four congruent sides.

RECTANGLE COROLLARY

A quadrilateral is a rectangle if and only if it has four right angles.

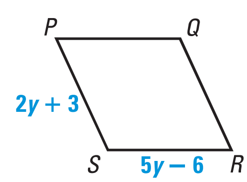
SQUARE COROLLARY

A quadrilateral is a square if and only if it is a rhombus and a rectangle.

You can use these corollaries to prove that a quadrilateral is a rhombus, rectangle, or square without proving first that the quadrilateral is a parallelogram.

GOAL 3 Using Properties of a Rhombus

In the diagram at the right, *PQRS* is a rhombus. What is the value of *y*?



SOLUTION

All four sides of a rhombus are congruent, so $RS = PS$.

$5y - 6 = 2y + 3$	Equate lengths of congruent sides.
$5y = 2y + 9$	Add 6 to each side.
$3y = 9$	Subtract $2y$ from each side.
$y = 3$	Divide each side by 3.

GOAL 2 USING DIAGONALS OF SPECIAL PARALLELOGRAMS

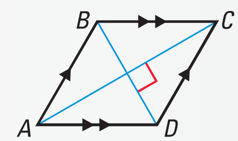
The following theorems are about diagonals of rhombuses and rectangles. You are asked to prove Theorems 6.12 and 6.13 in Exercises 51, 52, 59, and 60.

THEOREMS

THEOREM 6.11

A parallelogram is a rhombus if and only if its diagonals are perpendicular.

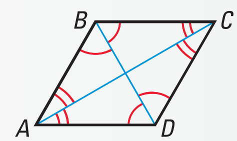
ABCD is a rhombus if and only if $\overline{AC} \perp \overline{BD}$.



THEOREM 6.12

A parallelogram is a rhombus if and only if each diagonal bisects a pair of opposite angles.

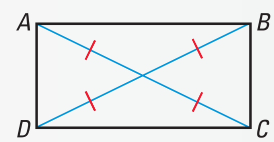
ABCD is a rhombus if and only if
$\overline{AC}$ bisects $\angle DAB$ and $\angle BCD$ and
$\overline{BD}$ bisects $\angle ADC$ and $\angle CBA$.



THEOREM 6.13

A parallelogram is a rectangle if and only if its diagonals are congruent.

ABCD is a rectangle if and only if $\overline{AC} \cong \overline{BD}$.



You can rewrite Theorem 6.11 as a conditional statement and its converse.

Conditional statement: If the diagonals of a parallelogram are perpendicular, then the parallelogram is a rhombus.

Converse: If a parallelogram is a rhombus, then its diagonals are perpendicular.

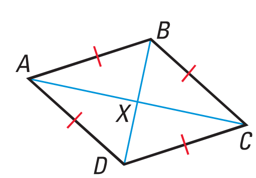
To prove the theorem, you must prove both statements.

EXAMPLE 4 *Proving Theorem 6.11*

Write a paragraph proof of the converse above.

GIVEN ▶ *ABCD* is a rhombus.

PROVE ▶ $\overline{AC} \perp \overline{BD}$



SOLUTION

Paragraph Proof *ABCD* is a rhombus, so $\overline{AB} \cong \overline{CB}$. Because *ABCD* is a parallelogram, its diagonals bisect each other so $\overline{AX} \cong \overline{CX}$ and $\overline{BX} \cong \overline{BX}$. Use the SSS Congruence Postulate to prove $\triangle AXB \cong \triangle CXB$, so $\angle AXB \cong \angle CXB$. Then, because $\overline{AC}$ and $\overline{BD}$ intersect to form congruent adjacent angles, $\overline{AC} \perp \overline{BD}$.

STUDENT HELP

INTERNET **HOMEWORK HELP**
Visit our Web site www.mcdougallittell.com for extra examples.

Proof

EXAMPLE 5 *Coordinate Proof of Theorem 6.11*

In Example 4, a paragraph proof was given for part of Theorem 6.11. Write a coordinate proof of the original conditional statement.

GIVEN ▶ $ABCD$ is a parallelogram, $\overline{AC} \perp \overline{BD}$.

PROVE ▶ $ABCD$ is a rhombus.

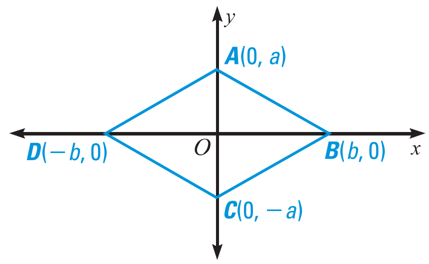
SOLUTION

Assign coordinates Because $\overline{AC} \perp \overline{BD}$, place $ABCD$ in the coordinate plane so $\overline{AC}$ and $\overline{BD}$ lie on the axes and their intersection is at the origin.

Let $(0, a)$ be the coordinates of A, and let $(b, 0)$ be the coordinates of B.

Because $ABCD$ is a parallelogram, the diagonals bisect each other and $OA = OC$. So, the coordinates of C are $(0, -a)$.

Similarly, the coordinates of D are $(-b, 0)$.



Find the lengths of the sides of $ABCD$. Use the Distance Formula.

$$AB = \sqrt{(b-0)^2 + (0-a)^2} = \sqrt{b^2 + a^2}$$

$$BC = \sqrt{(0-b)^2 + (-a-0)^2} = \sqrt{b^2 + a^2}$$

$$CD = \sqrt{(-b-0)^2 + [0-(-a)]^2} = \sqrt{b^2 + a^2}$$

$$DA = \sqrt{[0-(-b)]^2 + (a-0)^2} = \sqrt{b^2 + a^2}$$

▶ All of the side lengths are equal, so $ABCD$ is a rhombus.

FOCUS ON APPLICATIONS

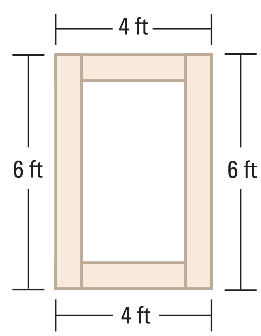
CARPENTRY If a screen door is not rectangular, you can use a piece of hardware called a *turnbuckle* to shorten the longer diagonal until the door is rectangular.

EXAMPLE 6 *Checking a Rectangle*

CARPENTRY You are building a rectangular frame for a theater set.

a. First, you nail four pieces of wood together, as shown at the right. What is the shape of the frame?

b. To make sure the frame is a rectangle, you measure the diagonals. One is 7 feet 4 inches and the other is 7 feet 2 inches. Is the frame a rectangle? Explain.



SOLUTION

a. Opposite sides are congruent, so the frame is a parallelogram.

b. The parallelogram is not a rectangle. If it were a rectangle, the diagonals would be congruent.

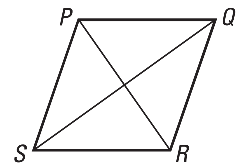
GUIDED PRACTICE

Vocabulary Check ✓

1. What is another name for an *equilateral quadrilateral*?

Concept Check ✓

2. Theorem 6.12 is a biconditional statement. Rewrite the theorem as a conditional statement and its converse, and tell what each statement means for parallelogram *PQRS*.



Skill Check ✓

Decide whether the statement is *sometimes*, *always*, or *never* true.

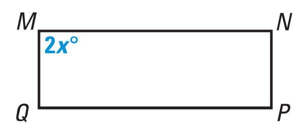
3. A rectangle is a parallelogram.
4. A parallelogram is a rhombus.
5. A rectangle is a rhombus.
6. A square is a rectangle.

Which of the following quadrilaterals have the given property?

7. All sides are congruent.
8. All angles are congruent.
9. The diagonals are congruent.
10. Opposite angles are congruent.

A. Parallelogram
B. Rectangle
C. Rhombus
D. Square

11. *MNPQ* is a rectangle. What is the value of x?



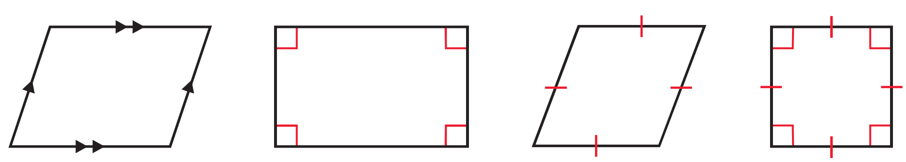
PRACTICE AND APPLICATIONS

STUDENT HELP
Extra Practice to help you master skills is on p. 814.

RECTANGLE **For any rectangle *ABCD*, decide whether the statement is *always*, *sometimes*, or *never* true. Draw a sketch and explain your answer.**

12. $\angle A \cong \angle B$
13. $\overline{AB} \cong \overline{BC}$
14. $\overline{AC} \cong \overline{BD}$
15. $\overline{AC} \perp \overline{BD}$

PROPERTIES **List each quadrilateral for which the statement is true.**



parallelogram **rectangle** **rhombus** **square**

16. It is equiangular.
17. It is equiangular and equilateral.
18. The diagonals are perpendicular.
19. Opposite sides are congruent.
20. The diagonals bisect each other.
21. The diagonals bisect opposite angles.

PROPERTIES **Sketch the quadrilateral and list everything you know about it.**

22. parallelogram *FGHI*
23. rhombus *PQRS*
24. square *ABCD*

STUDENT HELP
HOMEWORK HELP
Example 1: Exs. 12–15, 27–32
Example 2: Exs. 27–32, 51
Example 3: Exs. 33–43
Example 4: Exs. 44–52
Example 5: Exs. 55–60
Example 6: Exs. 61, 62

LOGICAL REASONING **Give another name for the quadrilateral.**

25. equiangular quadrilateral

26. regular quadrilateral

RHOMBUS **For any rhombus *ABCD*, decide whether the statement is *always*, *sometimes*, or *never* true. Draw a sketch and explain your answer.**

27. $\angle A \cong \angle C$

28. $\angle A \cong \angle B$

29. $\angle ABD \cong \angle CBD$

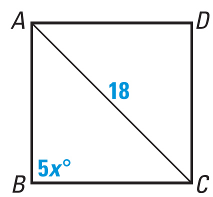
30. $\overline{AB} \cong \overline{BC}$

31. $\overline{AC} \cong \overline{BD}$

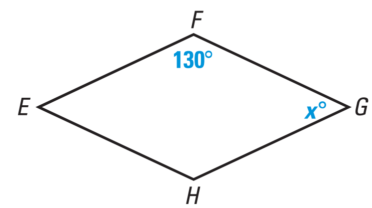
32. $\overline{AD} \cong \overline{CD}$

USING ALGEBRA **Find the value of *x*.**

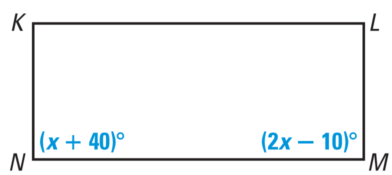
33. *ABCD* is a square.



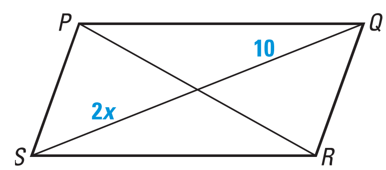
34. *EFGH* is a rhombus.



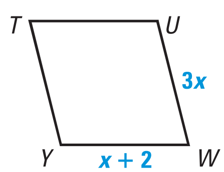
35. *KLMN* is a rectangle.



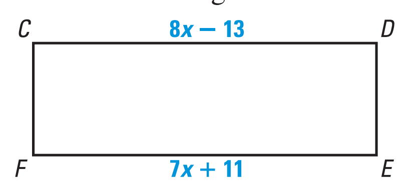
36. *PQRS* is a parallelogram.



37. *TUWY* is a rhombus.



38. *CDEF* is a rectangle.



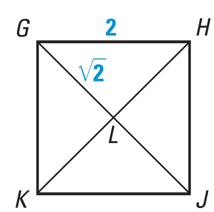
COMPLETING STATEMENTS ***GHJK*** **is a square with diagonals intersecting at *L*. Given that $GH = 2$ and $GL = \sqrt{2}$, complete the statement.**

39. $HK = \underline{\ ?\ }$

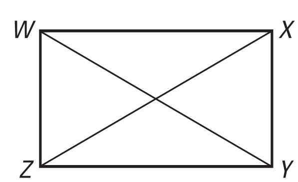
40. $m\angle KLJ = \underline{\ ?\ }$

41. $m\angle HJG = \underline{\ ?\ }$

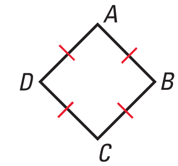
42. Perimeter of $\triangle HJK = \underline{\ ?\ }$



43. **USING ALGEBRA** *WXYZ* is a rectangle. The perimeter of $\triangle XYZ$ is 24. $XY + YZ = 5x - 1$ and $XZ = 13 - x$. Find *WY*.



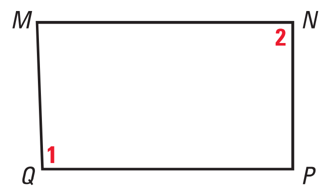
44. **LOGICAL REASONING** What additional information do you need to prove that $ABCD$ is a square?



PROOF **In Exercises 45 and 46, write any kind of proof.**

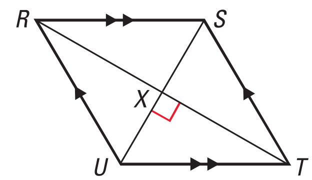
45. **GIVEN** ▶ $\overline{MN} \parallel \overline{PQ}$, $\angle 1 \not\cong \angle 2$

PROVE ▶ $\overline{MQ}$ is not parallel to $\overline{PN}$.

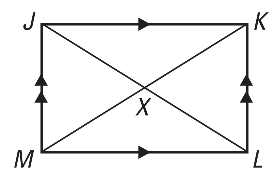


46. **GIVEN** ▶ $RSTU$ is a ▱, $\overline{SU} \perp \overline{RT}$

PROVE ▶ $\angle STR \cong \angle UTR$



47. **BICONDITIONAL STATEMENTS** Rewrite Theorem 6.13 as a conditional statement and its converse. Tell what each statement means for parallelogram $JKLM$.



LOGICAL REASONING Write the corollary as a conditional statement and its converse. Then explain why each statement is true.

48. Rhombus corollary

49. Rectangle corollary

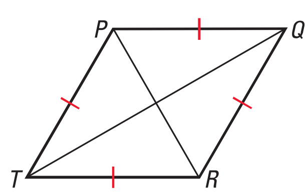
50. Square corollary

PROVING THEOREM 6.12 **Prove both conditional statements of Theorem 6.12.**

51. **GIVEN** ▶ $PQRT$ is a rhombus.

PROVE ▶ $\overline{PR}$ bisects $\angle TPQ$ and $\angle QRT$. $\overline{TQ}$ bisects $\angle PTR$ and $\angle RQP$.

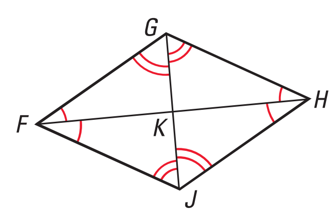
Plan for Proof To prove that $\overline{PR}$ bisects $\angle TPQ$ and $\angle QRT$, first prove that $\triangle PRQ \cong \triangle PRT$.



52. **GIVEN** ▶ $FGHJ$ is a parallelogram. $\overline{FH}$ bisects $\angle JFG$ and $\angle GHJ$. $\overline{JG}$ bisects $\angle FJH$ and $\angle HGF$.

PROVE ▶ $FGHJ$ is a rhombus.

Plan for Proof Prove $\triangle FHJ \cong \triangle FHG$ so $\overline{JH} \cong \overline{GH}$. Then use the fact that $\overline{JH} \cong \overline{FG}$ and $\overline{GH} \cong \overline{FJ}$.



CONSTRUCTION **Explain how to construct the figure using a straightedge and a compass. Use a definition or theorem from this lesson to explain why your method works.**

53. a rhombus that is not a square

54. a rectangle that is not a square

COORDINATE GEOMETRY **It is given that *PQRS* is a parallelogram. Graph ▱*PQRS*. Decide whether it is a *rectangle*, a *rhombus*, a *square*, or *none of the above*. Justify your answer using theorems about quadrilaterals.**

55. $P(3, 1)$
$Q(3, -3)$
$R(-2, -3)$
$S(-2, 1)$

56. $P(5, 2)$
$Q(1, 9)$
$R(-3, 2)$
$S(1, -5)$

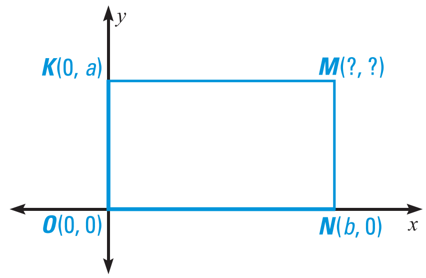
57. $P(-1, 4)$
$Q(-3, 2)$
$R(2, -3)$
$S(4, -1)$

58. $P(5, 2)$
$Q(2, 5)$
$R(-1, 2)$
$S(2, -1)$

COORDINATE PROOF OF THEOREM 6.13 **In Exercises 59 and 60, you will complete a coordinate proof of one conditional statement of Theorem 6.13.**

GIVEN ▶ *KMNO* is a rectangle.

PROVE ▶ $\overline{OM} \cong \overline{KN}$

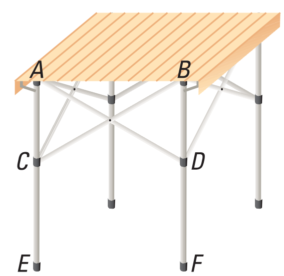


Because $\angle O$ is a right angle, place *KMNO* in the coordinate plane so *O* is at the origin, $\overline{ON}$ lies on the x-axis and $\overline{OK}$ lies on the y-axis. Let the coordinates of *K* be $(0, a)$ and let the coordinates of *N* be $(b, 0)$.

59. What are the coordinates of *M*? Explain your reasoning.

60. Use the Distance Formula to prove that $\overline{OM} \cong \overline{KN}$.

PORTABLE TABLE **The legs of the table shown at the right are all the same length. The cross braces are all the same length and bisect each other.**



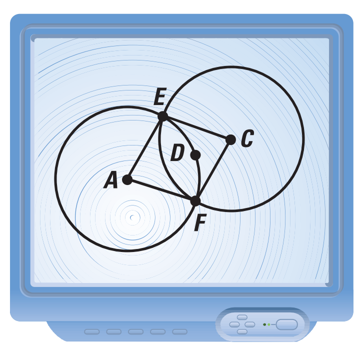
61. Show that the edge of the tabletop $\overline{AB}$ is perpendicular to legs $\overline{AE}$ and $\overline{BF}$.

62. Show that $\overline{AB}$ is parallel to $\overline{EF}$.

STUDENT HELP

SOFTWARE HELP
Visit our Web site www.mcdougallittell.com to see instructions for several software applications.

TECHNOLOGY **In Exercises 63–65, use geometry software.**



Draw a segment $\overline{AB}$ and a point *C* on the segment. Construct the midpoint *D* of $\overline{AB}$. Then hide $\overline{AB}$ and point *B* so only points *A*, *D*, and *C* are visible.

Construct two circles with centers *A* and *C* using the length $\overline{AD}$ as the radius of each circle. Label the points of intersection *E* and *F*. Draw $\overline{AE}$, $\overline{CE}$, $\overline{CF}$, and $\overline{AF}$.

63. What kind of shape is *AECF*? How do you know? What happens to the shape as you drag *A*? drag *C*?

64. Hide the circles and point *D*, and draw diagonals $\overline{EF}$ and $\overline{AC}$. Measure $\angle EAC$, $\angle FAC$, $\angle AEF$, and $\angle CEF$. What happens to the measures as you drag *A*? drag *C*?

65. Which theorem does this construction illustrate?

Test Preparation

66. **MULTIPLE CHOICE** In rectangle $ABCD$, if $AB = 7x - 3$ and $CD = 4x + 9$, then $x = \underline{\ ?\ }$.

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

67. **MULTIPLE CHOICE** In parallelogram $KLMN$, $KM = LN$, $m\angle KLM = 2xy$, and $m\angle LMN = 9x + 9$. Find the value of y.

(A) 9 (B) 5 (C) 18

(D) 10 (E) Cannot be determined.

68. *Writing* Explain why a parallelogram with one right angle is a rectangle.

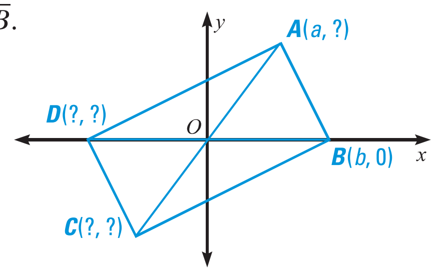
★ **Challenge**

COORDINATE PROOF OF THEOREM 6.13 **Complete the coordinate proof of one conditional statement of Theorem 6.13.**

GIVEN ▶ $ABCD$ is a parallelogram, $\overline{AC} \cong \overline{DB}$.

PROVE ▶ $ABCD$ is a rectangle.

Place $ABCD$ in the coordinate plane so $\overline{DB}$ lies on the x-axis and the diagonals intersect at the origin. Let the coordinates of B be $(b, 0)$ and let the x-coordinate of A be a as shown.



69. Explain why $OA = OB = OC = OD$.

70. Write the y-coordinate of A in terms of a and b. Explain your reasoning.

71. Write the coordinates of C and D in terms of a and b. Explain your reasoning.

72. Find and compare the slopes of the sides to prove that $ABCD$ is a rectangle.

EXTRA CHALLENGE
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MIXED REVIEW

USING THE SAS CONGRUENCE POSTULATE **Decide whether enough information is given to determine that $\triangle ABC \cong \triangle DEF$.** **(Review 4.3)**

73. $\angle A \cong \angle D$, $\overline{AB} \cong \overline{DE}$, $\overline{AC} \cong \overline{DF}$

74. $\overline{AB} \cong \overline{BC}$, $\overline{BC} \cong \overline{CA}$, $\angle A \cong \angle D$

75. $\angle B \cong \angle E$, $\overline{AC} \cong \overline{DF}$, $\overline{AB} \cong \overline{DE}$

76. $\overline{EF} \cong \overline{BC}$, $\overline{DF} \cong \overline{AB}$, $\angle A \cong \angle E$

77. $\angle C \cong \angle F$, $\overline{AC} \cong \overline{DF}$, $\overline{BC} \cong \overline{EF}$

78. $\angle B \cong \angle E$, $\overline{AB} \cong \overline{DE}$, $\overline{BC} \cong \overline{EF}$

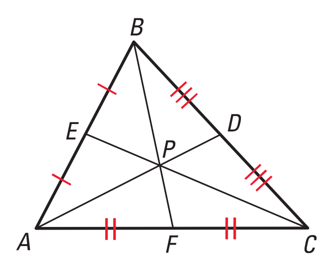
CONCURRENCY PROPERTY FOR MEDIANS **Use the information given in the diagram to fill in the blanks.** **(Review 5.3)**

79. $AP = 1$, $PD = \underline{\ ?\ }$

80. $PC = 6.6$, $PE = \underline{\ ?\ }$

81. $PB = 6$, $FB = \underline{\ ?\ }$

82. $AD = 39$, $PD = \underline{\ ?\ }$



83. **INDIRECT PROOF** Write an indirect proof to show that there is no quadrilateral with four acute angles. **(Review 6.1 for 6.5)**

6.5 Trapezoids and Kites

What you should learn

GOAL 1 Use properties of trapezoids.

GOAL 2 Use properties of kites.

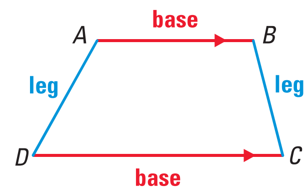
Why you should learn it

▼ To solve **real-life** problems, such as planning the layers of a layer cake in **Example 3**.

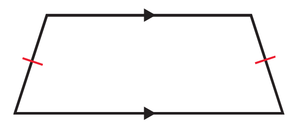
REAL LIFE

GOAL 1 USING PROPERTIES OF TRAPEZOIDS

A **trapezoid** is a quadrilateral with exactly one pair of parallel sides. The parallel sides are the **bases**. A trapezoid has two pairs of **base angles**. For instance, in trapezoid *ABCD*, $\angle D$ and $\angle C$ are one pair of base angles. The other pair is $\angle A$ and $\angle B$. The nonparallel sides are the **legs** of the trapezoid.



If the legs of a trapezoid are congruent, then the trapezoid is an **isosceles trapezoid**.



isosceles trapezoid

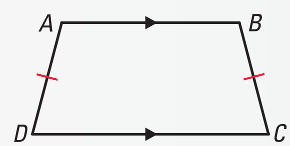
You are asked to prove the following theorems in the exercises.

THEOREMS

THEOREM 6.14

If a trapezoid is isosceles, then each pair of base angles is congruent.

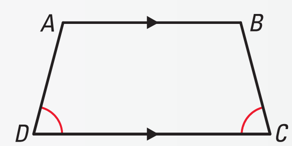
$\angle A \cong \angle B$, $\angle C \cong \angle D$



THEOREM 6.15

If a trapezoid has a pair of congruent base angles, then it is an isosceles trapezoid.

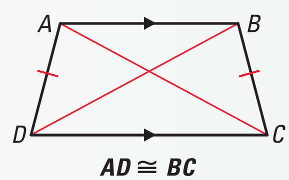
ABCD is an isosceles trapezoid.



THEOREM 6.16

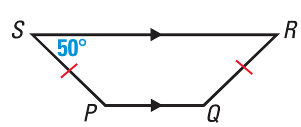
A trapezoid is isosceles if and only if its diagonals are congruent.

ABCD is isosceles if and only if $\overline{AC} \cong \overline{BD}$.



EXAMPLE 1 *Using Properties of Isosceles Trapezoids*

PQRS is an isosceles trapezoid. Find $m\angle P$, $m\angle Q$, and $m\angle R$.



SOLUTION *PQRS* is an isosceles trapezoid, so $m\angle R = m\angle S = 50°$. Because $\angle S$ and $\angle P$ are consecutive interior angles formed by parallel lines, they are supplementary. So, $m\angle P = 180° - 50° = 130°$, and $m\angle Q = m\angle P = 130°$.

STUDENT HELP

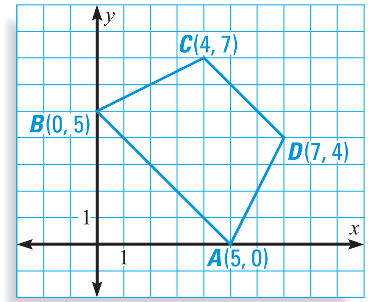
HOMEWORK HELP
Visit our Web site www.mcdougallittell.com for extra examples.

2 Using Properties of Trapezoids

Show that *ABCD* is a trapezoid.

SOLUTION

Compare the slopes of opposite sides.



The slope of $\overline{AB} = \frac{5-0}{0-5} = \frac{5}{-5} = -1$.

The slope of $\overline{CD} = \frac{4-7}{7-4} = \frac{-3}{3} = -1$.

The slopes of $\overline{AB}$ and $\overline{CD}$ are equal, so $\overline{AB} \parallel \overline{CD}$.

The slope of $\overline{BC} = \frac{7-5}{4-0} = \frac{2}{4} = \frac{1}{2}$.

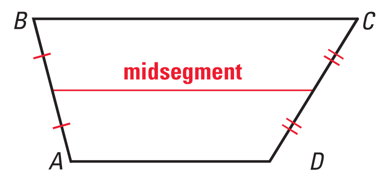
The slope of $\overline{AD} = \frac{4-0}{7-5} = \frac{4}{2} = 2$.

The slopes of $\overline{BC}$ and $\overline{AD}$ are not equal, so $\overline{BC}$ is not parallel to $\overline{AD}$.

▶ So, because $\overline{AB} \parallel \overline{CD}$ and $\overline{BC}$ is not parallel to $\overline{AD}$, *ABCD* is a trapezoid.

. . . .

The **midsegment** of a trapezoid is the segment that connects the midpoints of its legs. Theorem 6.17 is similar to the Midsegment Theorem for triangles. You will justify part of this theorem in Exercise 42. A proof appears on page 839.

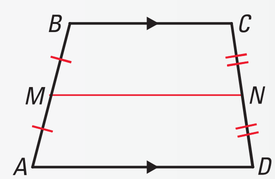


THEOREM

THEOREM 6.17 ***Midsegment Theorem for Trapezoids***

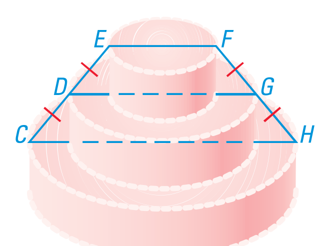
The midsegment of a trapezoid is parallel to each base and its length is one half the sum of the lengths of the bases.

$\overline{MN} \parallel \overline{AD}$, $\overline{MN} \parallel \overline{BC}$, $MN = \frac{1}{2}(AD + BC)$



3 Finding Midsegment Lengths of Trapezoids

LAYER CAKE A baker is making a cake like the one at the right. The top layer has a diameter of 8 inches and the bottom layer has a diameter of 20 inches. How big should the middle layer be?



SOLUTION

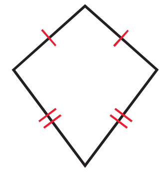
Use the Midsegment Theorem for Trapezoids.

$$DG = \frac{1}{2}(EF + CH) = \frac{1}{2}(8 + 20) = 14 \text{ inches}$$

The simplest of flying kites often use the geometric kite shape.

GOAL 2 USING PROPERTIES OF KITES

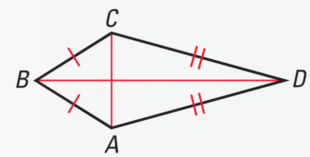
A **kite** is a quadrilateral that has two pairs of consecutive congruent sides, but opposite sides are not congruent. You are asked to prove Theorem 6.18 and Theorem 6.19 in Exercises 46 and 47.



THEOREMS ABOUT KITES

THEOREM 6.18

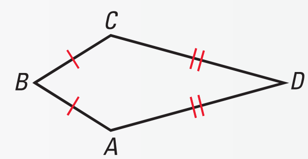
If a quadrilateral is a kite, then its diagonals are perpendicular.



$\overline{AC} \perp \overline{BD}$

THEOREM 6.19

If a quadrilateral is a kite, then exactly one pair of opposite angles are congruent.

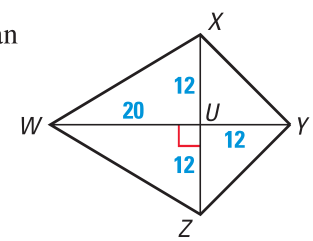


$\angle A \cong \angle C, \angle B \not\cong \angle D$

Using Algebra

4 Using the Diagonals of a Kite

WXYZ is a kite so the diagonals are perpendicular. You can use the Pythagorean Theorem to find the side lengths.



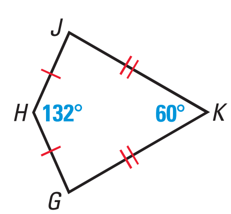
$$WX = \sqrt{20^2 + 12^2} \approx 23.32$$

$$XY = \sqrt{12^2 + 12^2} \approx 16.97$$

Because *WXYZ* is a kite, $WZ = WX \approx 23.32$ and $ZY = XY \approx 16.97$.

5 Angles of a Kite

Find $m\angle G$ and $m\angle J$ in the diagram at the right.



SOLUTION

GHJK is a kite, so $\angle G \cong \angle J$ and $m\angle G = m\angle J$.

$$2(m\angle G) + 132° + 60° = 360° \quad \text{Sum of measures of int. ⦞ of a quad. is 360°.}$$

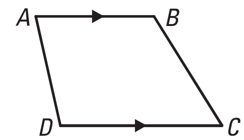
$$2(m\angle G) = 168° \quad \text{Simplify.}$$

$$m\angle G = 84° \quad \text{Divide each side by 2.}$$

▶ So, $m\angle J = m\angle G = 84°$.

GUIDED PRACTICE

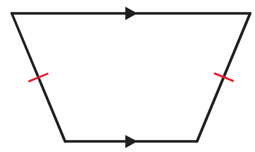
Vocabulary Check ✓ 1. Name the bases of trapezoid *ABCD*.



Concept Check ✓ 2. Explain why a rhombus is not a kite. Use the definition of a kite.

Skill Check ✓ **Decide whether the quadrilateral is a *trapezoid*, an *isosceles trapezoid*, a *kite*, or *none of these*.**

3.



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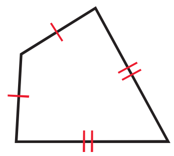

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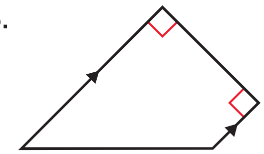
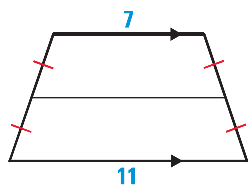


6. How can you prove that trapezoid *ABCD* in Example 2 is isosceles?

Find the length of the midsegment.

7.



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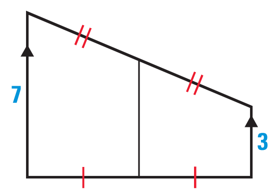
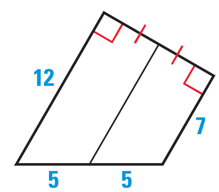


9.



PRACTICE AND APPLICATIONS

STUDENT HELP

Extra Practice to help you master skills is on p. 814.

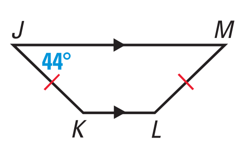
STUDYING A TRAPEZOID **Draw a trapezoid *PQRS* with $\overline{QR} \parallel \overline{PS}$. Identify the segments or angles of *PQRS* as *bases, consecutive sides, legs, diagonals, base angles,* or *opposite angles*.**

10. $\overline{QR}$ and $\overline{PS}$ **11.** $\overline{PQ}$ and $\overline{RS}$ **12.** $\overline{PQ}$ and $\overline{QR}$

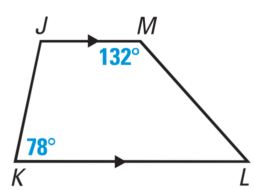
13. $\overline{QS}$ and $\overline{PR}$ **14.** $\angle Q$ and $\angle S$ **15.** $\angle S$ and $\angle P$

FINDING ANGLE MEASURES **Find the angle measures of *JKLM*.**

16.



17.



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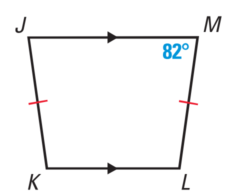
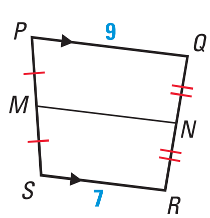


STUDENT HELP

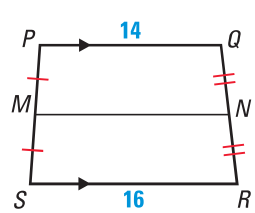
HOMEWORK HELP
Example 1: Exs. 16–18
Example 2: Exs. 34, 37, 38, 48–50
Example 3: Exs. 19–24, 35, 39
Example 4: Exs. 28–30
Example 5: Exs. 31–33

FINDING MIDSEGMENTS **Find the length of the midsegment $\overline{MN}$.**

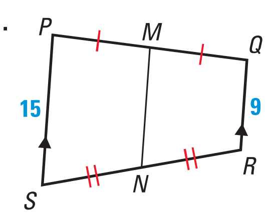
19.



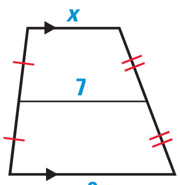
20.

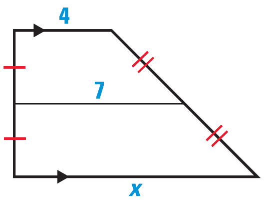


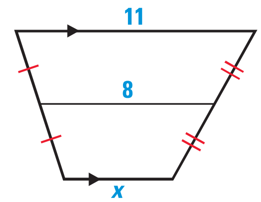
21.



USING ALGEBRA Find the value of x.

22. 

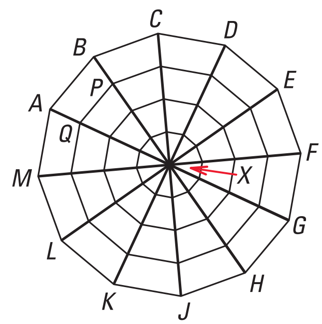
23. 

24. 

FOCUS ON APPLICATIONS

REAL LIFE **WEBS** The spider web above is called an orb web. Although it looks like concentric polygons, the spider actually followed a spiral path to spin the web.

CONCENTRIC POLYGONS **In the diagram, *ABCDEFGHJKLM* is a regular dodecagon, $\overline{AB} \parallel \overline{PQ}$, and X is equidistant from the vertices of the dodecagon.**

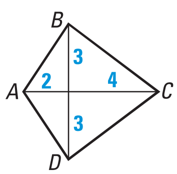


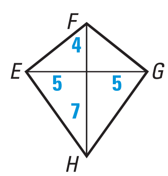
25. Are you given enough information to prove that $ABPQ$ is isosceles? Explain your reasoning.

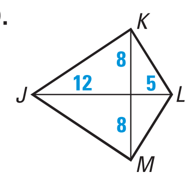
26. What is the measure of $\angle AXB$?

27. What is the measure of each interior angle of $ABPQ$?

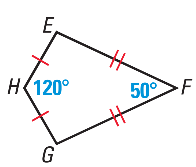
USING ALGEBRA **What are the lengths of the sides of the kite? Give your answer to the nearest hundredth.**

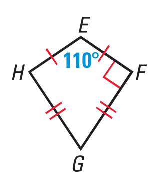
28. 

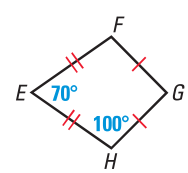
29. 

30. 

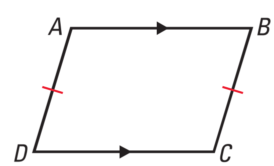
ANGLES OF KITES ***EFGH* is a kite. What is $m\angle G$?**

31. 

32. 

33. 

34. **ERROR ANALYSIS** A student says that parallelogram $ABCD$ is an isosceles trapezoid because $\overline{AB} \parallel \overline{DC}$ and $\overline{AD} \cong \overline{BC}$. Explain what is wrong with this reasoning.



35. **CRITICAL THINKING** The midsegment of a trapezoid is 5 inches long. What are possible lengths of the bases?

36. **COORDINATE GEOMETRY** Determine whether the points $A(4, 5)$, $B(-3, 3)$, $C(-6, -13)$, and $D(6, -2)$ are the vertices of a kite. Explain your answer.

TRAPEZOIDS **Determine whether the given points represent the vertices of a trapezoid. If so, is the trapezoid isosceles? Explain your reasoning.**

37. $A(-2, 0)$, $B(0, 4)$, $C(5, 4)$, $D(8, 0)$

38. $E(1, 9)$, $F(4, 2)$, $G(5, 2)$, $H(8, 9)$

FOCUS ON CAREERS

CAKE DESIGNERS design cakes for many occasions, including weddings, birthdays, anniversaries, and graduations.

CAREER LINK
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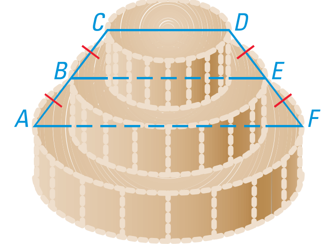
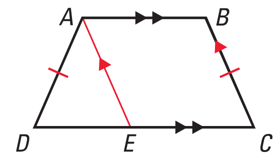
39. **LAYER CAKE** The top layer of the cake has a diameter of 10 inches. The bottom layer has a diameter of 22 inches. What is the diameter of the middle layer?



40. **PROVING THEOREM 6.14** Write a proof of Theorem 6.14.

GIVEN ▶ $ABCD$ is an isosceles trapezoid.
$\overline{AB} \parallel \overline{DC}$, $\overline{AD} \cong \overline{BC}$

PROVE ▶ $\angle D \cong \angle C$, $\angle DAB \cong \angle B$

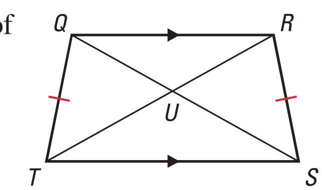


Plan for Proof To show $\angle D \cong \angle C$, first draw $\overline{AE}$ so $ABCE$ is a parallelogram. Then show $\overline{BC} \cong \overline{AE}$, so $\overline{AE} \cong \overline{AD}$ and $\angle D \cong \angle AED$. Finally, show $\angle D \cong \angle C$. To show $\angle DAB \cong \angle B$, use the consecutive interior angles theorem and substitution.

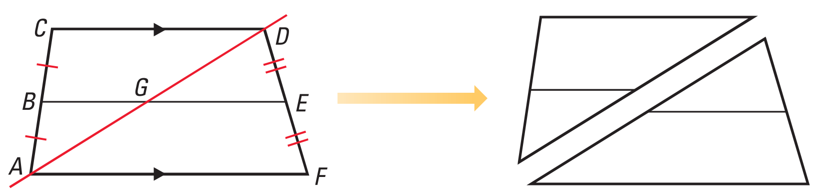
41. **PROVING THEOREM 6.16** Write a proof of one conditional statement of Theorem 6.16.

GIVEN ▶ $TQRS$ is an isosceles trapezoid.
$\overline{QR} \parallel \overline{TS}$ and $\overline{QT} \cong \overline{RS}$

PROVE ▶ $\overline{TR} \cong \overline{SQ}$



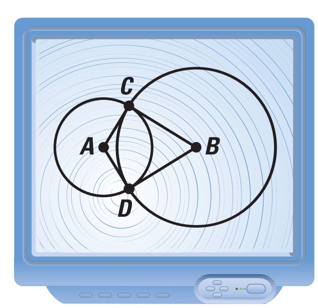
42. **JUSTIFYING THEOREM 6.17** In the diagram below, $\overline{BG}$ is the midsegment of $\triangle ACD$ and $\overline{GE}$ is the midsegment of $\triangle ADF$. Explain why the midsegment of trapezoid $ACDF$ is parallel to each base and why its length is one half the sum of the lengths of the bases.



STUDENT HELP

SOFTWARE HELP
Visit our Web site www.mcdougallittell.com to see instructions for several software applications.

USING TECHNOLOGY **In Exercises 43–45, use geometry software.**
Draw points A, B, C and segments $\overline{AC}$ and $\overline{BC}$. Construct a circle with center A and radius AC. Construct a circle with center B and radius BC. Label the other intersection of the circles D. Draw $\overline{BD}$ and $\overline{AD}$.

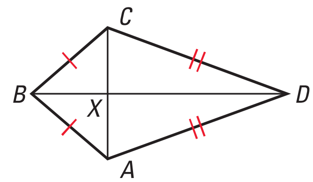


43. What kind of shape is $ACBD$? How do you know? What happens to the shape as you drag A? drag B? drag C?

44. Measure $\angle ACB$ and $\angle ADB$. What happens to the angle measures as you drag A, B, or C?

45. Which theorem does this construction illustrate?

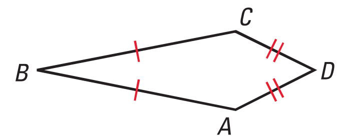
46. **PROVING THEOREM 6.18** Write a two-column proof of Theorem 6.18.



GIVEN ▶ $\overline{AB} \cong \overline{CB}$, $\overline{AD} \cong \overline{CD}$

PROVE ▶ $\overline{AC} \perp \overline{BD}$

47. **PROVING THEOREM 6.19** Write a paragraph proof of Theorem 6.19.

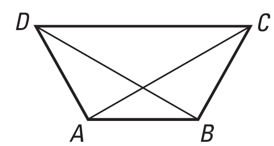


GIVEN ▶ $ABCD$ is a kite with $\overline{AB} \cong \overline{CB}$ and $\overline{AD} \cong \overline{CD}$.

PROVE ▶ $\angle A \cong \angle C$, $\angle B \not\cong \angle D$

Plan for Proof First show that $\angle A \cong \angle C$. Then use an indirect argument to show $\angle B \not\cong \angle D$: If $\angle B \cong \angle D$, then $ABCD$ is a parallelogram. But opposite sides of a parallelogram are congruent. This contradicts the definition of a kite.

TRAPEZOIDS Decide whether you are given enough information to conclude that *ABCD* is an isosceles trapezoid. Explain your reasoning.



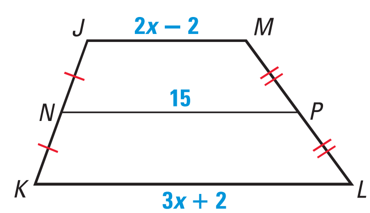
48. $\overline{AB} \parallel \overline{DC}$
$\overline{AD} \cong \overline{BC}$
$\overline{AD} \cong \overline{AB}$

49. $\overline{AB} \parallel \overline{DC}$
$\overline{AC} \cong \overline{BD}$
$\angle A \not\cong \angle C$

50. $\angle A \cong \angle B$
$\angle D \cong \angle C$
$\angle A \not\cong \angle C$

Test Preparation

51. **MULTIPLE CHOICE** In the trapezoid at the right, $NP = 15$. What is the value of x?



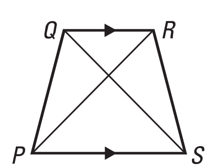
Ⓐ 2 Ⓑ 3 Ⓒ 4
Ⓓ 5 Ⓔ 6

52. **MULTIPLE CHOICE** Which one of the following can a trapezoid have?

Ⓐ congruent bases
Ⓑ diagonals that bisect each other
Ⓒ exactly two congruent sides
Ⓓ a pair of congruent opposite angles
Ⓔ exactly three congruent angles

★ Challenge

53. **PROOF** Prove one direction of Theorem 6.16: If the diagonals of a trapezoid are congruent, then the trapezoid is isosceles.



GIVEN ▶ $PQRS$ is a trapezoid.
$\overline{QR} \parallel \overline{PS}$, $\overline{PR} \cong \overline{SQ}$

PROVE ▶ $\overline{QP} \cong \overline{RS}$

Plan for Proof Draw a perpendicular segment from Q to $\overline{PS}$ and label the intersection M. Draw a perpendicular segment from R to $\overline{PS}$ and label the intersection N. Prove that $\triangle QMS \cong \triangle RNP$. Then prove that $\triangle QPS \cong \triangle RSP$.

EXTRA CHALLENGE
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MIXED REVIEW

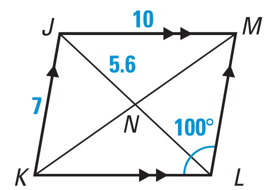
CONDITIONAL STATEMENTS **Rewrite the statement in if-then form.** **(Review 2.1)**

54. A scalene triangle has no congruent sides.

55. A kite has perpendicular diagonals.

56. A polygon is a pentagon if it has five sides.

FINDING MEASUREMENTS **Use the diagram to find the side length or angle measure.** **(Review 6.2 for 6.6)**



57. LN **58.** KL **59.** ML

60. JL **61.** $m\angle JML$ **62.** $m\angle MJK$

PARALLELOGRAMS **Determine whether the given points represent the vertices of a parallelogram. Explain your answer.** **(Review 6.3 for 6.6)**

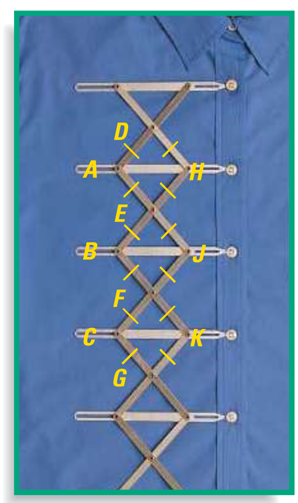
63. $A(-2, 8), B(5, 8), C(2, 0), D(-5, 0)$

64. $P(4, -3), Q(9, -1), R(8, -6), S(3, -8)$

QUIZ 2

Self-Test for Lessons 6.4 and 6.5

1. **POSITIONING BUTTONS** The tool at the right is used to decide where to put buttons on a shirt. The tool is stretched to fit the length of the shirt, and the pointers show where to put the buttons. Why are the pointers always evenly spaced? (*Hint:* You can prove that $\overline{HJ} \cong \overline{JK}$ if you know that $\triangle JFK \cong \triangle HEJ$.) **(Lesson 6.4)**



Determine whether the given points represent the vertices of a *rectangle*, a *rhombus*, a *square*, a *trapezoid*, or a *kite*. **(Lessons 6.4, 6.5)**

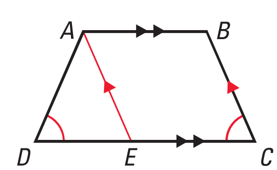
2. $P(2, 5), Q(-4, 5), R(2, -7), S(-4, -7)$

3. $A(-3, 6), B(0, 9), C(3, 6), D(0, -10)$

4. $J(-5, 6), K(-4, -2), L(4, -1), M(3, 7)$

5. $P(-5, -3), Q(1, -2), R(6, 3), S(7, 9)$

6. **PROVING THEOREM 6.15** Write a proof of Theorem 6.15.



GIVEN ▶ $ABCD$ is a trapezoid with $\overline{AB} \parallel \overline{DC}$.
$\angle D \cong \angle C$

PROVE ▶ $\overline{AD} \cong \overline{BC}$

Plan for Proof Draw $\overline{AE}$ so $ABCE$ is a parallelogram. Use the Transitive Property of Congruence to show $\angle AED \cong \angle D$. Then $\overline{AD} \cong \overline{AE}$, so $\overline{AD} \cong \overline{BC}$. **(Lesson 6.5)**

6.6 Special Quadrilaterals

What you should learn

GOAL 1 Identify special quadrilaterals based on limited information.

GOAL 2 Prove that a quadrilateral is a special type of quadrilateral, such as a rhombus or a trapezoid.

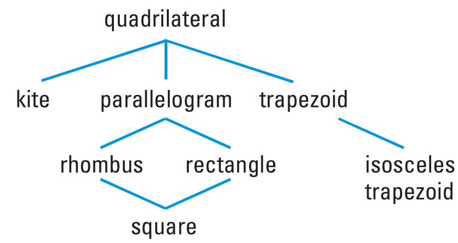
Why you should learn it

▼ To understand and describe **real-world** shapes such as gem facets in **Exs. 42 and 43.**

REAL LIFE

GOAL 1 SUMMARIZING PROPERTIES OF QUADRILATERALS

In this chapter, you have studied the seven special types of quadrilaterals at the right. Notice that each shape has all the properties of the shapes linked above it. For instance, squares have the properties of rhombuses, rectangles, parallelograms, and quadrilaterals.



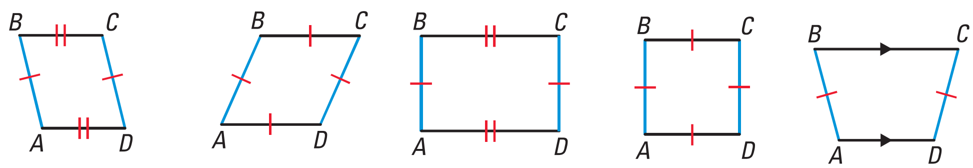
EXAMPLE 1 *Identifying Quadrilaterals*

Quadrilateral *ABCD* has at least one pair of opposite sides congruent. What kinds of quadrilaterals meet this condition?

SOLUTION

There are many possibilities.

PARALLELOGRAM | **RHOMBUS** | **RECTANGLE** | **SQUARE** | **ISOSCELES TRAPEZOID**



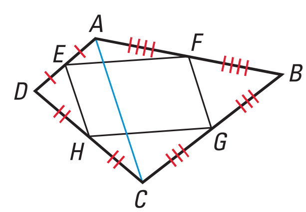
Opposite sides are congruent. | **All sides are congruent.** | **Opposite sides are congruent.** | **All sides are congruent.** | **Legs are congruent.**

EXAMPLE 2 *Connecting Midpoints of Sides*

When you join the midpoints of the sides of any quadrilateral, what special quadrilateral is formed? Why?

SOLUTION

Let *E*, *F*, *G*, and *H* be the midpoints of the sides of any quadrilateral, *ABCD*, as shown.



If you draw $\overline{AC}$, the Midsegment Theorem for triangles says $\overline{FG} \parallel \overline{AC}$ and $\overline{EH} \parallel \overline{AC}$, so $\overline{FG} \parallel \overline{EH}$. Similar reasoning shows that $\overline{EF} \parallel \overline{HG}$.

▶ So, by definition, *EFGH* is a parallelogram.

GOAL 2 PROOF WITH SPECIAL QUADRILATERALS

When you want to prove that a quadrilateral has a specific shape, you can use either the definition of the shape as in Example 2, *or* you can use a theorem.

CONCEPT SUMMARY — PROVING QUADRILATERALS ARE RHOMBUSES

You have learned three ways to prove that a quadrilateral is a rhombus.

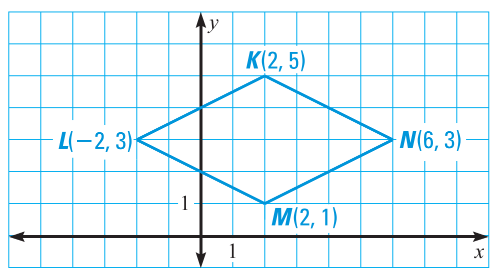
1. You can use the definition and show that the quadrilateral is a *parallelogram* that has four congruent sides. It is easier, however, to use the Rhombus Corollary and simply show that all four sides of the quadrilateral are congruent.
2. Show that the quadrilateral is a parallelogram *and* that the diagonals are perpendicular. (*Theorem 6.11*)
3. Show that the quadrilateral is a parallelogram *and* that each diagonal bisects a pair of opposite angles. (*Theorem 6.12*)

STUDENT HELP

Look Back
For help with proving a quadrilateral is a parallelogram, see pp. 338–341.

EXAMPLE 3 *Proving a Quadrilateral is a Rhombus*

Show that $KLMN$ is a rhombus.



SOLUTION You can use any of the three ways described in the concept summary above. For instance, you could show that opposite sides have the same slope and that the diagonals are perpendicular. Another way, shown below, is to prove that all four sides have the same length.

$$LM = \sqrt{[2-(-2)]^2 + (1-3)^2} = \sqrt{4^2 + (-2)^2} = \sqrt{20}$$

$$NK = \sqrt{(2-6)^2 + (5-3)^2} = \sqrt{(-4)^2 + 2^2} = \sqrt{20}$$

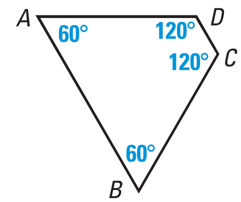
$$MN = \sqrt{(6-2)^2 + (3-1)^2} = \sqrt{4^2 + 2^2} = \sqrt{20}$$

$$KL = \sqrt{(-2-2)^2 + (3-5)^2} = \sqrt{(-4)^2 + (-2)^2} = \sqrt{20}$$

So, because $LM = NK = MN = KL$, $KLMN$ is a rhombus.

4 *Identifying a Quadrilateral*

What type of quadrilateral is *ABCD*? Explain your reasoning.



SOLUTION

$\angle A$ and $\angle D$ are supplementary, but $\angle A$ and $\angle B$ are not. So, $\overline{AB} \parallel \overline{DC}$ but $\overline{AD}$ is not parallel to $\overline{BC}$. By definition, *ABCD* is a trapezoid. Because base angles are congruent, *ABCD* is an isosceles trapezoid.

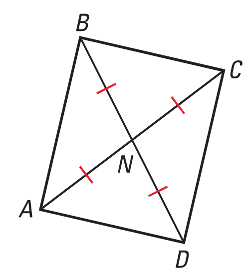
5 *Identifying a Quadrilateral*

The diagonals of quadrilateral *ABCD* intersect at point *N* to produce four congruent segments: $\overline{AN} \cong \overline{BN} \cong \overline{CN} \cong \overline{DN}$. What type of quadrilateral is *ABCD*? Prove that your answer is correct.

SOLUTION

Draw a diagram:

Draw the diagonals as described. Then connect the endpoints to draw quadrilateral *ABCD*.



Make a conjecture:

Quadrilateral *ABCD* looks like a rectangle.

Proof

Prove your conjecture:

GIVEN ▶ $\overline{AN} \cong \overline{BN} \cong \overline{CN} \cong \overline{DN}$

PROVE ▶ *ABCD* is a rectangle.

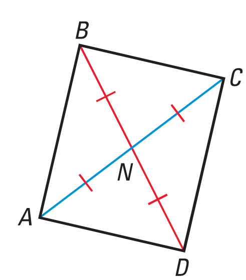
Paragraph Proof Because you are given information about the diagonals, show that *ABCD* is a parallelogram with congruent diagonals.

First prove that *ABCD* is a parallelogram.

Because $\overline{BN} \cong \overline{DN}$ and $\overline{AN} \cong \overline{CN}$, $\overline{BD}$ and $\overline{AC}$ bisect each other. Because the diagonals of *ABCD* bisect each other, *ABCD* is a parallelogram.

Then prove that the diagonals of *ABCD* are congruent.

From the given you can write $BN = AN$ and $DN = CN$ so, by the Addition Property of Equality, $BN + DN = AN + CN$. By the Segment Addition Postulate, $BD = BN + DN$ and $AC = AN + CN$ so, by substitution, $BD = AC$.



So, $\overline{BD} \cong \overline{AC}$.

▶ *ABCD* is a parallelogram with congruent diagonals, so *ABCD* is a rectangle.

GUIDED PRACTICE

Concept Check ✓

1. In Example 2, explain how to prove that $\overline{EF} \parallel \overline{HG}$.

Skill Check ✓

Copy the chart. Put an X in the box if the shape *always* has the given property.

	Property	▱	Rectangle	Rhombus	Square	Kite	Trapezoid
2.	Both pairs of opp. sides are $\parallel$.	?	?	?	?	?	?
3.	Exactly 1 pair of opp. sides are $\parallel$.	?	?	?	?	?	?
4.	Diagonals are $\perp$.	?	?	?	?	?	?
5.	Diagonals are $\cong$.	?	?	?	?	?	?
6.	Diagonals bisect each other.	?	?	?	?	?	?

7. Which quadrilaterals can you form with four sticks of the same length? You must attach the sticks at their ends and cannot bend or break any of them.

PRACTICE AND APPLICATIONS

STUDENT HELP

Extra Practice to help you master skills is on p. 814.

PROPERTIES OF QUADRILATERALS **Copy the chart. Put an X in the box if the shape *always* has the given property.**

	Property	▱	Rectangle	Rhombus	Square	Kite	Trapezoid
8.	Both pairs of opp. sides are $\cong$.	?	?	?	?	?	?
9.	Exactly 1 pair of opp. sides are $\cong$.	?	?	?	?	?	?
10.	All sides are $\cong$.	?	?	?	?	?	?
11.	Both pairs of opp. $\angle$s are $\cong$.	?	?	?	?	?	?
12.	Exactly 1 pair of opp. $\angle$s are $\cong$.	?	?	?	?	?	?
13.	All $\angle$s are $\cong$.	?	?	?	?	?	?

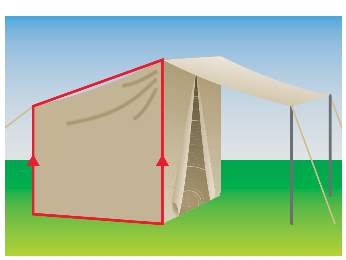
FOCUS ON APPLICATIONS

REAL LIFE

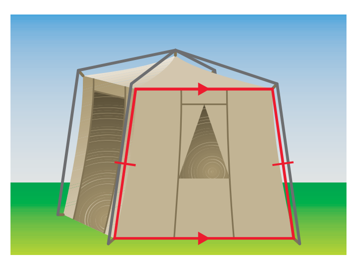
Tents are designed differently for different climates. For example, winter tents are designed to shed snow. Desert tents can have flat roofs because they don't need to shed rain.

TENT SHAPES **What kind of special quadrilateral is the red shape?**

14.



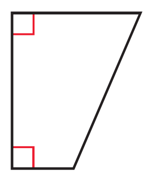
15.



> **STUDENT HELP**
>
> **HOMEWORK HELP**
> **Example 1:** Exs. 8–24, 30–35
> **Example 2:** Exs. 14–18, 42–44
> **Example 3:** Exs. 25–29, 36–41
> **Example 4:** Exs. 14–18, 42, 43
> **Example 5:** Exs. 45–47

IDENTIFYING QUADRILATERALS **Identify the special quadrilateral. Use the most specific name.**

16.



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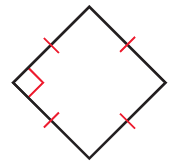
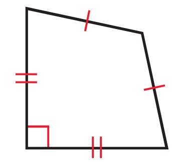
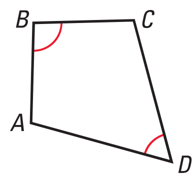


18.



IDENTIFYING QUADRILATERALS **What kinds of quadrilaterals meet the conditions shown? *ABCD* is not drawn to scale.**

19.



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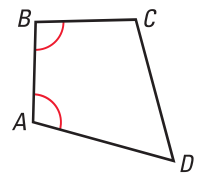

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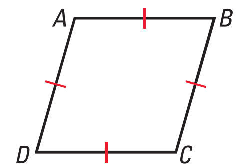

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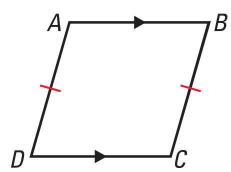

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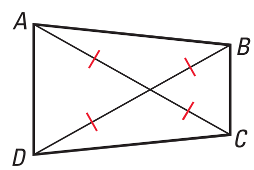
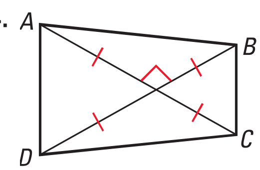


24.



> **STUDENT HELP**
>
> **Study Tip**
> See the summaries for parallelograms and rhombuses on pp. 340 and 365, and the list of postulates and theorems on pp. 828–837. You can refer to your summaries as you do the rest of the exercises.

DESCRIBING METHODS OF PROOF **Summarize the ways you have learned to prove that a quadrilateral is the given special type of quadrilateral.**

25. kite

26. square

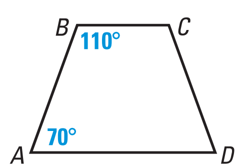
27. rectangle

28. trapezoid

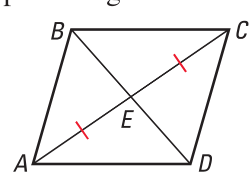
29. isosceles trapezoid

DEVELOPING PROOF **Which two segments or angles must be congruent to enable you to prove *ABCD* is the given quadrilateral? Explain your reasoning. There may be more than one right answer.**

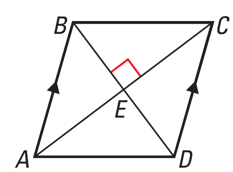
30. isosceles trapezoid



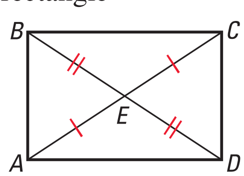
31. parallelogram



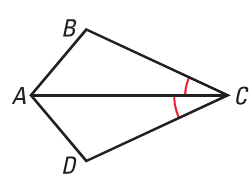
32. rhombus



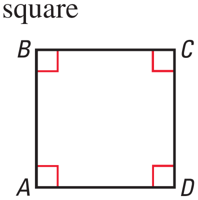
33. rectangle



34. kite



35. square



QUADRILATERALS **What kind of quadrilateral is *PQRS*? Justify your answer.**

36. $P(0, 0)$, $Q(0, 2)$, $R(5, 5)$, $S(2, 0)$

37. $P(1, 1)$, $Q(5, 1)$, $R(4, 8)$, $S(2, 8)$

38. $P(2, 1)$, $Q(7, 1)$, $R(7, 7)$, $S(2, 5)$

39. $P(0, 7)$, $Q(4, 8)$, $R(5, 2)$, $S(1, 1)$

40. $P(1, 7)$, $Q(5, 9)$, $R(8, 3)$, $S(4, 1)$

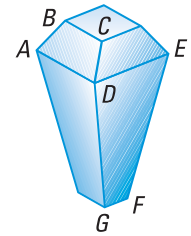
41. $P(5, 1)$, $Q(9, 6)$, $R(5, 11)$, $S(1, 6)$

FOCUS ON CAREERS

GEMOLOGISTS analyze the cut of a gem when determining its value.

CAREER LINK
www.mcdougallittell.com

GEM CUTTING **In Exercises 42 and 43, use the following information.**
There are different ways of cutting gems to enhance the beauty of the jewel. One of the earliest shapes used for diamonds is called the *table cut*, as shown at the right. Each face of a cut gem is called a *facet*.



42. $\overline{BC} \parallel \overline{AD}$, $\overline{AB}$ and $\overline{DC}$ are not parallel. What shape is the facet labeled $ABCD$?

43. $\overline{DE} \parallel \overline{GF}$, $\overline{DG}$ and $\overline{EF}$ are congruent, but not parallel. What shape is the facet labeled $DEFG$?

44. **JUSTIFYING A CONSTRUCTION** Look back at the *Perpendicular to a Line* construction on page 130. Explain why this construction works.

DRAWING QUADRILATERALS **Draw $\overline{AC}$ and $\overline{BD}$ as described. What special type of quadrilateral is *ABCD*? Prove that your answer is correct.**

45. $\overline{AC}$ and $\overline{BD}$ bisect each other, but they are not perpendicular or congruent.

46. $\overline{AC}$ and $\overline{BD}$ bisect each other. $\overline{AC} \perp \overline{BD}$, $\overline{AC} \not\cong \overline{BD}$

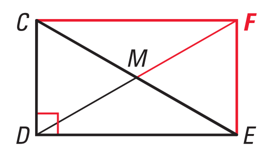
47. $\overline{AC} \perp \overline{BD}$, and $\overline{AC}$ bisects $\overline{BD}$. $\overline{BD}$ does not bisect $\overline{AC}$.

48. **LOGICAL REASONING** $EFGH$, $GHJK$, and $JKLM$ are all parallelograms. If $\overline{EF}$ and $\overline{LM}$ are not collinear, what kind of quadrilateral is $EFLM$? Prove that your answer is correct.

STUDENT HELP

HOMEWORK HELP
Visit our Web site www.mcdougallittell.com for help with Exs. 45–47.

49. **PROOF** Prove that the median of a right triangle is one half the length of the hypotenuse.



GIVEN ▶ $\angle CDE$ is a right angle. $\overline{CM} \cong \overline{EM}$

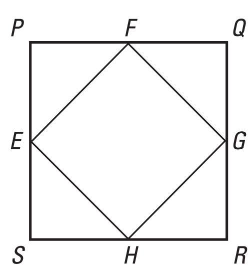
PROVE ▶ $\overline{DM} \cong \overline{CM}$

Plan for Proof First draw $\overline{CF}$ and $\overline{EF}$ so $CDEF$ is a rectangle. (How?)

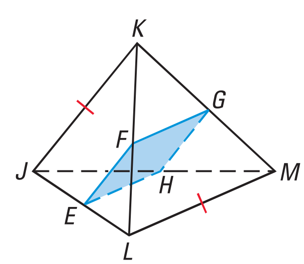
50. **PROOF** Use facts about angles to prove that the quadrilateral in Example 5 is a rectangle. (*Hint:* Let $x°$ be the measure of $\angle ABN$. Find the measures of the other angles in terms of x.)

PROOF **What special type of quadrilateral is *EFGH*? Prove that your answer is correct.**

51. **GIVEN** ▶ $PQRS$ is a square. E, F, G, and H are midpoints of the sides of the square.

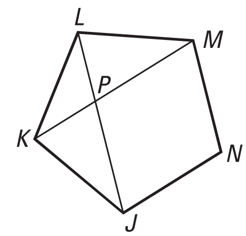


52. **GIVEN** ▶ $\overline{JK} \cong \overline{LM}$, E, F, G, and H are the midpoints of $\overline{JL}$, $\overline{KL}$, $\overline{KM}$, and $\overline{JM}$.



Test Preparation

53. **MULTI-STEP PROBLEM** Copy the diagram. $JKLMN$ is a regular pentagon. You will identify $JPMN$.



a. What kind of triangle is $\triangle JKL$? Use $\triangle JKL$ to prove that $\angle LJN \cong \angle JLM$.

b. List everything you know about the interior angles of $JLMN$. Use these facts to prove that $\overline{JL} \parallel \overline{NM}$.

c. Reasoning similar to parts (a) and (b) shows that $\overline{KM} \parallel \overline{JN}$. Based on this and the result from part (b), what kind of shape is $JPMN$?

d. *Writing* Is $JPMN$ a rhombus? Justify your answer.

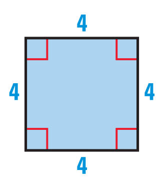
★ **Challenge**

54. **PROOF** $\overline{AC}$ and $\overline{BD}$ intersect each other at N. $\overline{AN} \cong \overline{BN}$ and $\overline{CN} \cong \overline{DN}$, but $\overline{AC}$ and $\overline{BD}$ do not bisect each other. Draw $\overline{AC}$ and $\overline{BD}$, and $ABCD$. What special type of quadrilateral is $ABCD$? Write a plan for a proof of your answer.

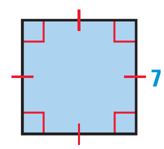
MIXED REVIEW

FINDING AREA **Find the area of the figure.** (Review 1.7 for 6.7)

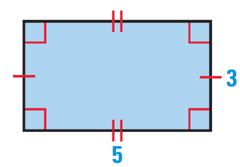
55.



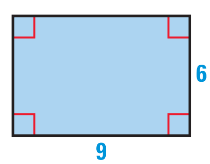
56.



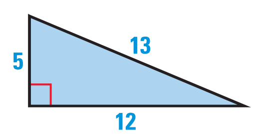
57.



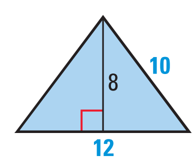
58.



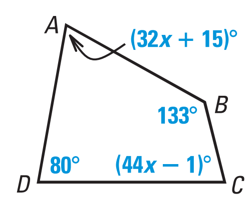
59.



60.



USING ALGEBRA **In Exercises 61 and 62, use the diagram at the right.** (Review 6.1)

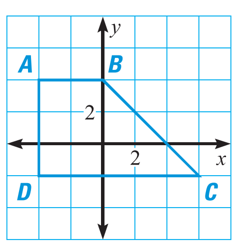


61. What is the value of x?

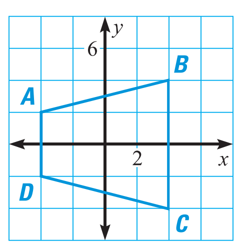
62. What is $m\angle A$? Use your result from Exercise 61.

FINDING THE MIDSEGMENT **Find the length of the midsegment of the trapezoid.** (Review 6.5 for 6.7)

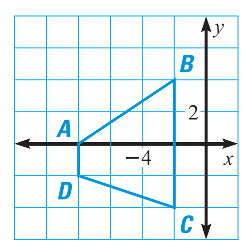
63.



64.



65.



6.7 Areas of Triangles and Quadrilaterals

What you should learn

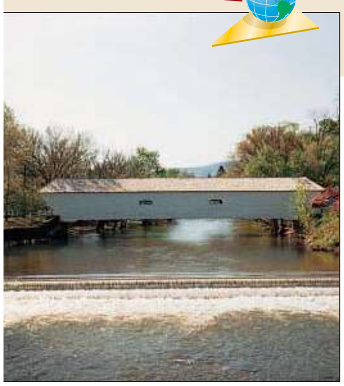
GOAL 1 Find the areas of squares, rectangles, parallelograms, and triangles.

GOAL 2 Find the areas of trapezoids, kites, and rhombuses, as applied in **Example 6**.

Why you should learn it

▼ To find areas of **real-life** surfaces, such as the roof of the covered bridge in **Exs. 48 and 49**.

REAL LIFE



GOAL 1 USING AREA FORMULAS

You can use the postulates below to prove several area theorems.

AREA POSTULATES

POSTULATE 22 ***Area of a Square Postulate***

The area of a square is the square of the length of its side, or $A = s^2$.

POSTULATE 23 ***Area Congruence Postulate***

If two polygons are congruent, then they have the same area.

POSTULATE 24 ***Area Addition Postulate***

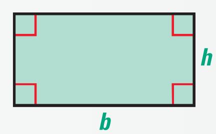
The area of a region is the sum of the areas of its nonoverlapping parts.

AREA THEOREMS

THEOREM 6.20 ***Area of a Rectangle***

The area of a rectangle is the product of its base and height.

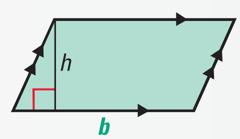
$$A = bh$$



THEOREM 6.21 ***Area of a Parallelogram***

The area of a parallelogram is the product of a base and its corresponding height.

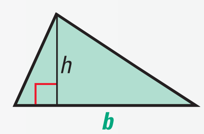
$$A = bh$$



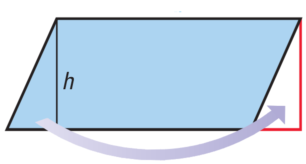
THEOREM 6.22 ***Area of a Triangle***

The area of a triangle is one half the product of a base and its corresponding height.

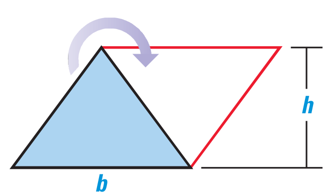
$$A = \frac{1}{2}bh$$



You can justify the area formulas for triangles and parallelograms as follows.



The area of a parallelogram is the area of a rectangle with the same base and height.



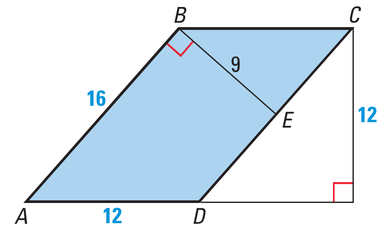
The area of a triangle is half the area of a parallelogram with the same base and height.

STUDENT HELP

Study Tip
To find the area of a parallelogram or triangle, you can use any side as the base. But be sure you measure the height of an altitude that is perpendicular to the base you have chosen.

EXAMPLE 1 Using the Area Theorems

Find the area of $\square ABCD$.



SOLUTION

Method 1 Use $\overline{AB}$ as the base. So, $b = 16$ and $h = 9$.

Area $= bh = 16(9) = 144$ square units.

Method 2 Use $\overline{AD}$ as the base. So, $b = 12$ and $h = 12$.

Area $= bh = 12(12) = 144$ square units.

Notice that you get the same area with either base.

Using Algebra

EXAMPLE 2 Finding the Height of a Triangle

Rewrite the formula for the area of a triangle in terms of h. Then use your formula to find the height of a triangle that has an area of 12 and a base length of 6.

SOLUTION

Rewrite the area formula so h is alone on one side of the equation.

$A = \frac{1}{2}bh$ Formula for the area of a triangle

$2A = bh$ Multiply both sides by 2.

$\frac{2A}{b} = h$ Divide both sides by b.

Substitute 12 for A and 6 for b to find the height of the triangle.

$$h = \frac{2A}{b} = \frac{2(12)}{6} = 4$$

▶ The height of the triangle is 4.

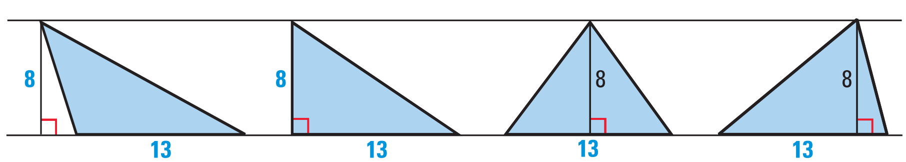
EXAMPLE 3 Finding the Height of a Triangle

A triangle has an area of 52 square feet and a base of 13 feet. Are all triangles with these dimensions congruent?

SOLUTION

Using the formula from Example 2, the height is $h = \frac{2(52)}{13} = 8$ feet.

There are many triangles with these dimensions. Some are shown below.



STUDENT HELP

Study Tip
Notice that the altitude of a triangle can be outside the triangle.

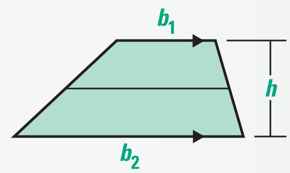
GOAL 2 Areas of Trapezoids, Kites, and Rhombuses

THEOREMS

THEOREM 6.23 ***Area of a Trapezoid***

The area of a trapezoid is one half the product of the height and the sum of the bases.

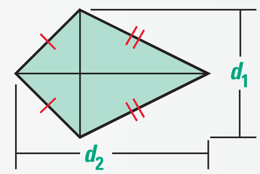
$$A = \frac{1}{2}h(b_1 + b_2)$$



THEOREM 6.24 ***Area of a Kite***

The area of a kite is one half the product of the lengths of its diagonals.

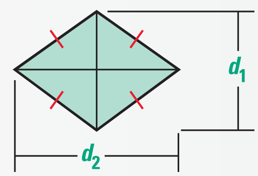
$$A = \frac{1}{2}d_1d_2$$



THEOREM 6.25 ***Area of a Rhombus***

The area of a rhombus is equal to one half the product of the lengths of the diagonals.

$$A = \frac{1}{2}d_1d_2$$

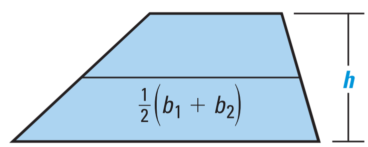


STUDENT HELP

Look Back
Remember that the length of the midsegment of a trapezoid is the average of the lengths of the bases. (p. 357)

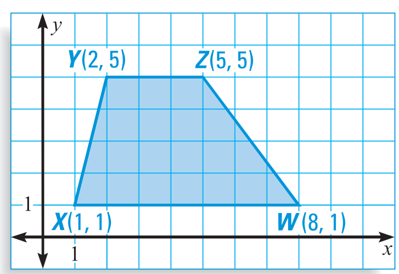
You will justify Theorem 6.23 in Exercises 58 and 59. You may find it easier to remember the theorem this way.

Area = **Length of Midsegment** • **Height**



EXAMPLE 4 *Finding the Area of a Trapezoid*

Find the area of trapezoid *WXYZ*.



SOLUTION

The height of *WXYZ* is $h = 5 - 1 = 4$.

Find the lengths of the bases.

$b_1 = YZ = 5 - 2 = 3$

$b_2 = XW = 8 - 1 = 7$

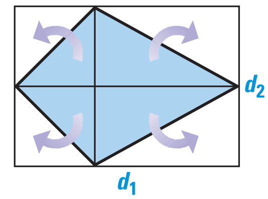
Substitute 4 for h, 3 for b_1, and 7 for b_2 to find the area of the trapezoid.

$A = \frac{1}{2}h(b_1 + b_2)$	Formula for area of a trapezoid
$= \frac{1}{2}(4)(3 + 7)$	Substitute.
$= 20$	Simplify.

▶ The area of trapezoid *WXYZ* is 20 square units.

The diagram at the right justifies the formulas for the areas of kites and rhombuses.

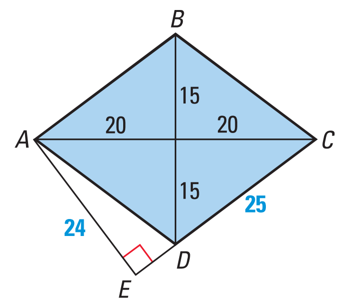
The diagram shows that the area of a kite is half the area of the rectangle whose length and width are the lengths of the diagonals of the kite. The same is true for a rhombus.



$A = \frac{1}{2}d_1d_2$

EXAMPLE 5 Finding the Area of a Rhombus

Use the information given in the diagram to find the area of rhombus *ABCD*.



SOLUTION

Method 1 Use the formula for the area of a rhombus. $d_1 = BD = 30$ and $d_2 = AC = 40$.

$$A = \frac{1}{2}d_1d_2$$
$$= \frac{1}{2}(30)(40)$$
$$= 600 \text{ square units}$$

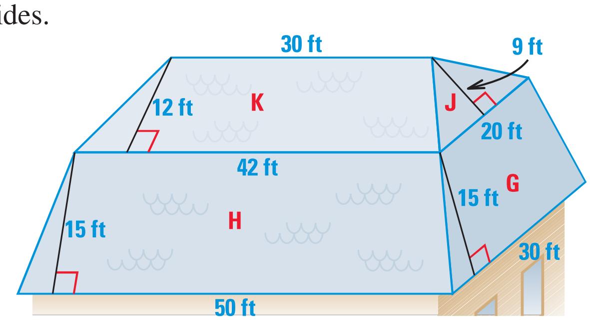
Method 2 Use the formula for the area of a parallelogram. $b = 25$ and $h = 24$.

$$A = bh = 25(24) = 600 \text{ square units}$$

EXAMPLE 6 Finding Areas

REAL LIFE

ROOF Find the area of the roof. *G*, *H*, and *K* are trapezoids and *J* is a triangle. The hidden back and left sides of the roof are the same as the front and right sides.



SOLUTION

Area of $J = \frac{1}{2}(20)(9) = 90 \text{ ft}^2$

Area of $H = \frac{1}{2}(15)(42 + 50) = 690 \text{ ft}^2$

Area of $G = \frac{1}{2}(15)(20 + 30) = 375 \text{ ft}^2$

Area of $K = \frac{1}{2}(12)(30 + 42) = 432 \text{ ft}^2$

The roof has two congruent faces of each type.

Total Area $= 2(90 + 375 + 690 + 432) = 3174$

▶ The total area of the roof is 3174 square feet.

STUDENT HELP

Study Tip
To check that the answer is reasonable, approximate each trapezoid by a rectangle. The area of *H* should be less than 50 • 15, but more than 40 • 15.

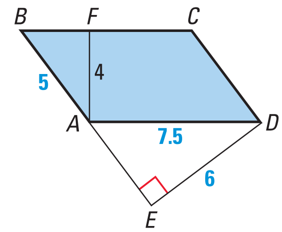
GUIDED PRACTICE

Vocabulary Check ✓

1. What is the *midsegment* of a trapezoid?

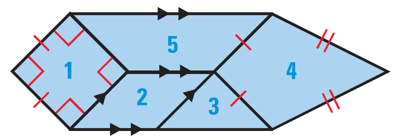
Concept Check ✓

2. If you use AB as the base to find the area of $\square ABCD$ shown at the right, what should you use as the height?



Skill Check ✓

Match the region with a formula for its area. Use each formula exactly once.



3. Region 1 — Ⓐ $A = s^2$

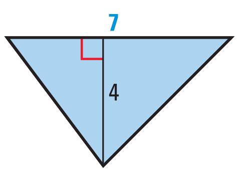
4. Region 2 — Ⓑ $A = \frac{1}{2}d_1d_2$

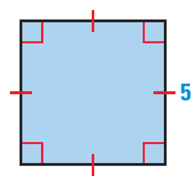
5. Region 3 — Ⓒ $A = \frac{1}{2}bh$

6. Region 4 — Ⓓ $A = \frac{1}{2}h(b_1 + b_2)$

7. Region 5 — Ⓔ $A = bh$

Find the area of the polygon.

8.


9.


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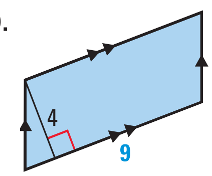

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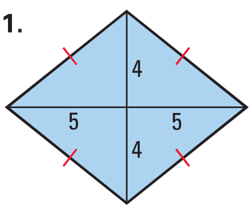
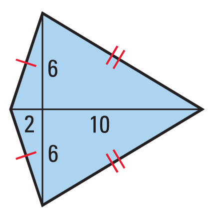
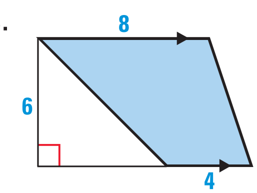


12.


13.


PRACTICE AND APPLICATIONS

STUDENT HELP

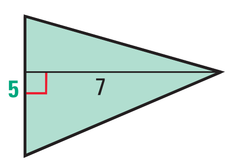
Extra Practice to help you master skills is on p. 814.

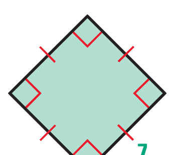
STUDENT HELP

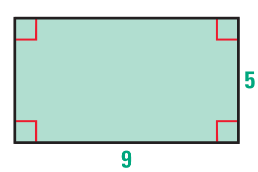
HOMEWORK HELP
Example 1: Exs. 14–19, 41–47
Example 2: Exs. 26–31

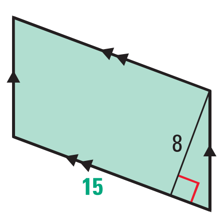
continued on p. 377

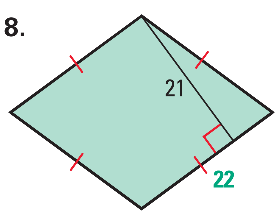
FINDING AREA **Find the area of the polygon.**

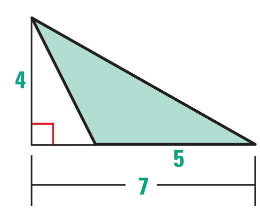
14.


15.


16.


17.


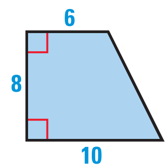
18.


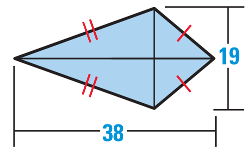
19.


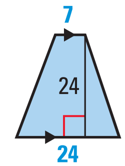
STUDENT HELP

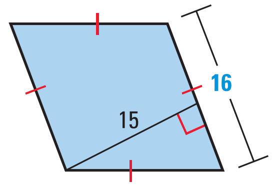
HOMEWORK HELP
continued from p. 376
Example 3: Exs. 26–28, 39, 40
Example 4: Exs. 32–34
Example 5: Exs. 20–25, 44
Example 6: Exs. 35–38, 48–52

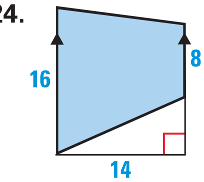
FINDING AREA Find the area of the polygon.

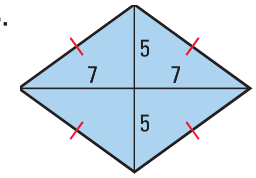
20.


21.


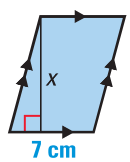
22.


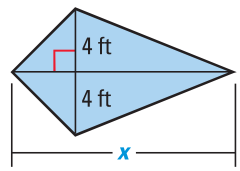
23.


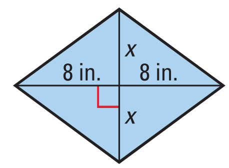
24.


25.


USING ALGEBRA Find the value of x.

26. $A = 63 \text{ cm}^2$


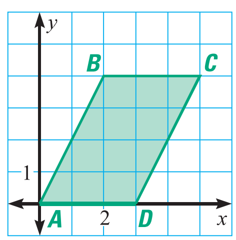
27. $A = 48 \text{ ft}^2$


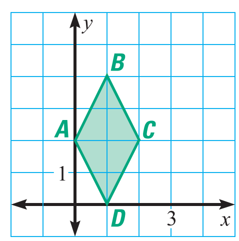
28. $A = 48 \text{ in.}^2$


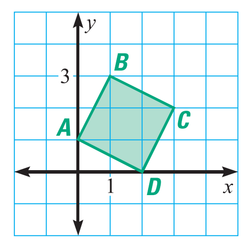
REWRITING FORMULAS Rewrite the formula for the area of the polygon in terms of the given variable. Use the formulas on pages 372 and 374.

29. triangle, b **30.** kite, d_1 **31.** trapezoid, b_1

FINDING AREA Find the area of quadrilateral *ABCD*.

32.


33.


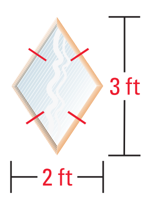
34.


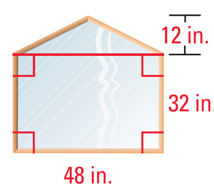
FOCUS ON APPLICATIONS

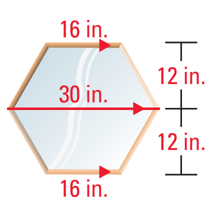
INSULATION Insulation makes a building more energy efficient. The ability of a material to insulate is called its *R*-value. Many windows have an *R*-value of 1. Adobe has an *R*-value of 11.9.

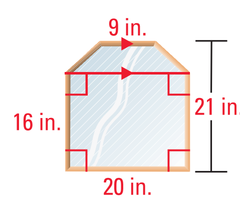
APPLICATION LINK
www.mcdougallittell.com

ENERGY CONSERVATION The total area of a building's windows affects the cost of heating or cooling the building. Find the area of the window.

35.


36.


37.


38.


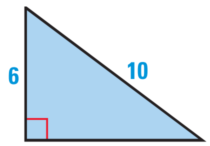
STUDENT HELP

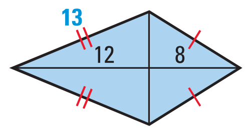
Study Tip
Remember that two polygons are congruent if their corresponding angles and sides are congruent.

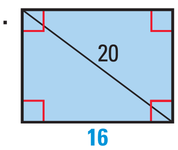
39. **LOGICAL REASONING** Are all parallelograms with an area of 24 square feet and a base of 6 feet congruent? Explain.

40. **LOGICAL REASONING** Are all rectangles with an area of 24 square feet and a base of 6 feet congruent? Explain.

USING THE PYTHAGOREAN THEOREM **Find the area of the polygon.**

41.


42.


43.


44. **LOGICAL REASONING** What happens to the area of a kite if you double the length of one of the diagonals? if you double the length of both diagonals?

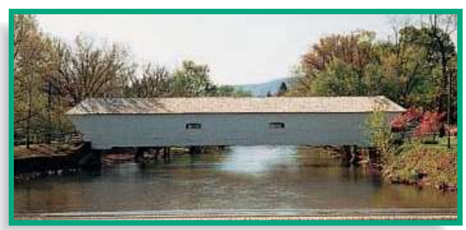
PARADE FLOATS **You are decorating a float for a parade. You estimate that, on average, a carnation will cover 3 square inches, a daisy will cover 2 square inches, and a chrysanthemum will cover 4 square inches. About how many flowers do you need to cover the shape on the float?**

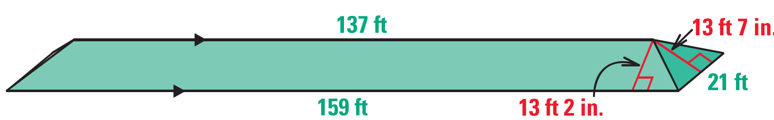
45. Carnations: 2 ft by 5 ft rectangle

46. Daisies: trapezoid ($b_1 = 5$ ft, $b_2 = 3$ ft, $h = 2$ ft)

47. Chrysanthemums: triangle ($b = 3$ ft, $h = 8$ ft)

BRIDGES **In Exercises 48 and 49, use the following information.**
The town of Elizabethton, Tennessee, restored the roof of this covered bridge with cedar shakes, a kind of rough wooden shingle. The shakes vary in width, but the average width is about 10 inches. So, on average, each shake protects a 10 inch by 10 inch square of roof.





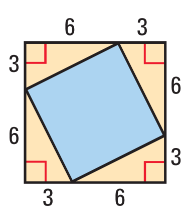
48. In the diagram of the roof, the hidden back and left sides are the same as the front and right sides. What is the total area of the roof?

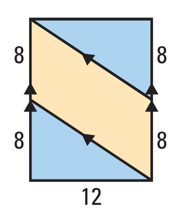
49. Estimate the number of shakes needed to cover the roof.

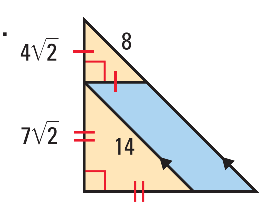
FOCUS ON PEOPLE

MARK CANDELARIA
When Mark Candelaria restored historic buildings in Scottsdale, Arizona, he calculated the areas of the walls and floors that needed to be replaced.

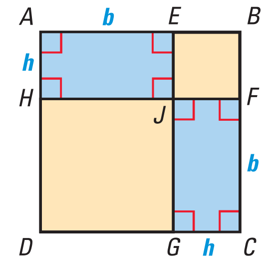
AREAS **Find the areas of the blue and yellow regions.**

50.


51.


52.


JUSTIFYING THEOREM 6.20 **In Exercises 53–57, you will justify the formula for the area of a rectangle. In the diagram, *AEJH* and *JFCG* are congruent rectangles with base length *b* and height *h*.**



53. What kind of shape is *EBFJ*? *HJGD*? Explain.

54. What kind of shape is *ABCD*? How do you know?

55. Write an expression for the length of a side of *ABCD*. Then write an expression for the area of *ABCD*.

56. Write expressions for the areas of *EBFJ* and *HJGD*.

STUDENT HELP

Look Back
For help with squaring binomial expressions, see p. 798.

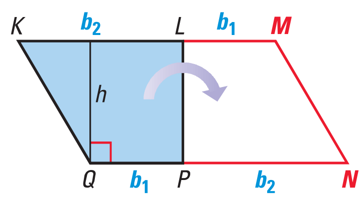
57. Substitute your answers from Exercises 55 and 56 into the following equation.

Let A = the area of *AEJH*. Solve the equation to find an expression for A.
Area of *ABCD* = Area of *HJGD* + Area of *EBFJ* + 2(Area of *AEJH*)

JUSTIFYING THEOREM 6.23 **Exercises 58 and 59 illustrate two ways to prove Theorem 6.23. Use the diagram to write a plan for a proof.**

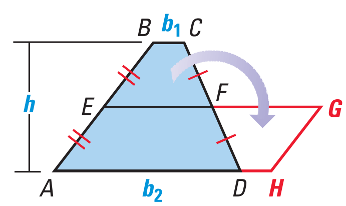
58. GIVEN ▶ *LPQK* is a trapezoid as shown. $LPQK \cong PLMN$.

PROVE ▶ The area of *LPQK* is $\frac{1}{2}h(b_1 + b_2)$.



59. GIVEN ▶ *ABCD* is a trapezoid as shown. $EBCF \cong GHDF$.

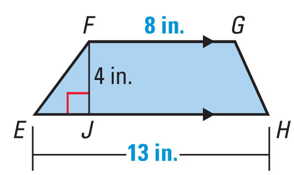
PROVE ▶ The area of *ABCD* is $\frac{1}{2}h(b_1 + b_2)$.



Test Preparation

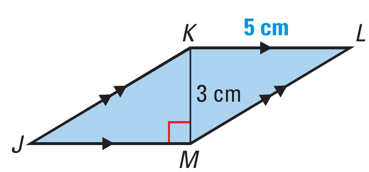
60. MULTIPLE CHOICE What is the area of trapezoid *EFGH*?

Ⓐ 25 in.^{2} Ⓑ 416 in.^{2}
Ⓒ 84 in.^{2} Ⓓ 42 in.^{2}
Ⓔ 68 in.^{2}



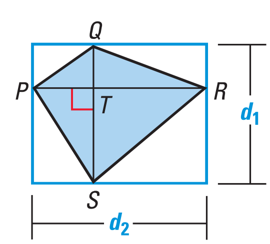
61. MULTIPLE CHOICE What is the area of parallelogram *JKLM*?

Ⓐ 12 cm^2 Ⓑ 15 cm^2
Ⓒ 18 cm^2 Ⓓ 30 cm^2
Ⓔ 40 cm^2



★ Challenge

62. *Writing* Explain why the area of *any* quadrilateral with perpendicular diagonals is $A = \frac{1}{2}d_1d_2$, where d_1 and d_2 are the lengths of the diagonals.



EXTRA CHALLENGE
www.mcdougallittell.com